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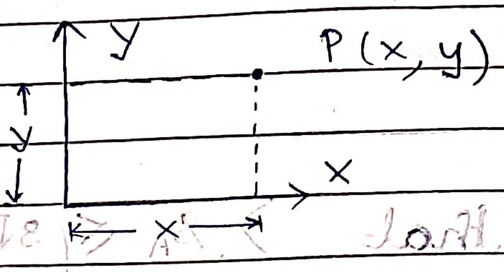
Straight Lines

Coordinate Systems —

1) Cartesian :



2) Parametric :

$\boxed{y = g(t)}$, $\boxed{x = f(t)}$

3) Polar :

$(x, y) \longrightarrow (r, \theta)$; $\boxed{\begin{array}{l} x = r c_\theta \\ y = r s_\theta \end{array}}$

Straight Line —

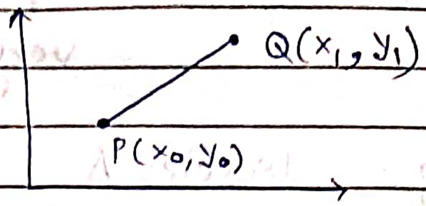
It is a curve s.t. line segment joining 2 pts on it, lies wholly on it.

Coordinate Geometry —

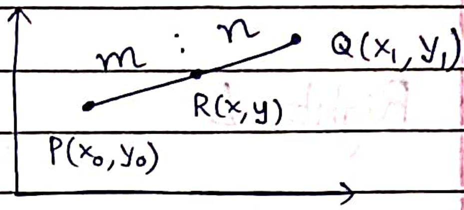
Basic Coordinates –

1) Dist. formula :

$$\boxed{PQ = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2}}$$



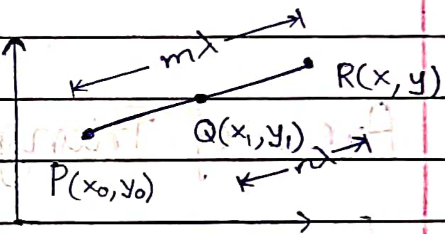
2) Section Formula :



Internal : (PR < PQ) $\left(\frac{RP}{RQ} = \frac{m}{n}\right)$

$$\boxed{(x, y) \equiv \left(\frac{mx_1+nx_0}{m+n}, \frac{my_1+ny_0}{m+n}\right)}$$

External : (PQ < PR) $\left(\frac{RP}{RQ} = \frac{m}{n}\right)$

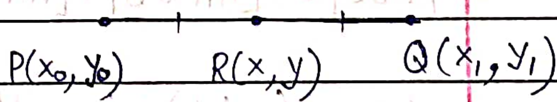


$$\boxed{(x, y) \equiv \left(\frac{mx_1-nx_0}{m-n}, \frac{my_1-ny_0}{m-n}\right)}$$

☆ If during calc. ratio m/n comes out to be (–ve) $\Rightarrow$ External Division

3) Midpt. Formula :

$$\boxed{(x, y) = \left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}\right)}$$



☆ In questions, assume ratio of division (λ:1)

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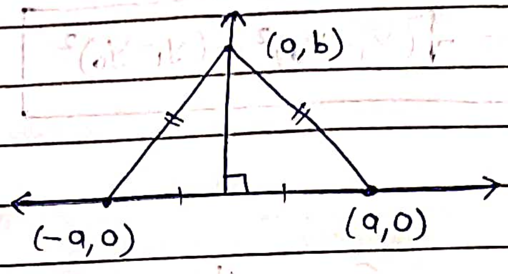
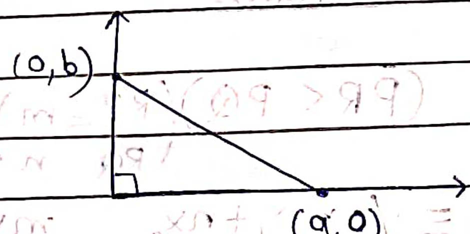
☆ In questions, assume coordinates axes very CAREFULLY.

Eg: Isoceles Δ

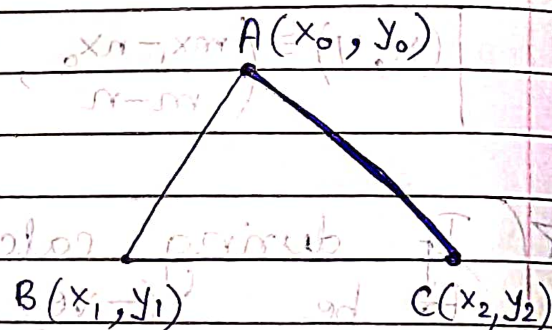


Right Δ



4) Area of Triangle:

$$\Delta = \left(\frac{1}{2}\right)\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

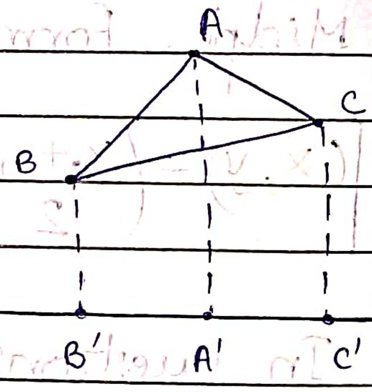


Proof:

$$[ABC] = [AA'B'B] + [AA'C'C] - [BB'C'C]$$

$$= \left(\frac{x_0 - x_1}{2}\right)(y_0 + y_1) + \left(\frac{x_2 - x_0}{2}\right)(y_0 + y_2)$$

$$- \left(\frac{x_2 - x_1}{2}\right)(y_1 + y_2)$$



5) Area of Quadrilateral :

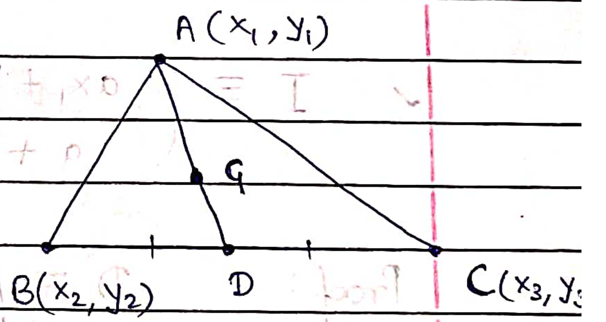
$$\text{Area} = \left(\frac{1}{2}\right)\left[\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix} + \begin{vmatrix} x_4 & x_1 \\ y_4 & y_1 \end{vmatrix}\right]$$

☆ 3 pts collinear $\Longleftrightarrow$ Area of $\Delta = 0$

4 pts. collinear $\Longleftrightarrow$ Area of quad. $= 0$

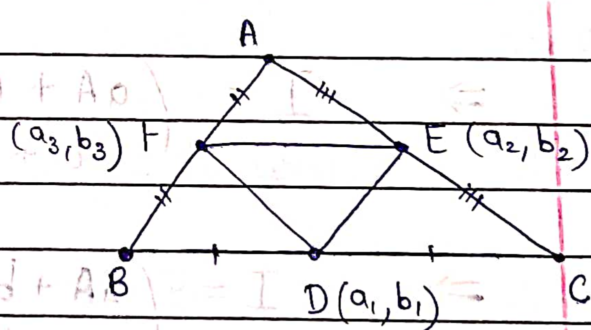
6) Terms related to Δ :

i) Centroid : $\cap$ of Medians



✓ $\left(\frac{AG}{GD}\right) = 2$

✓ $G \equiv \left(\frac{\Sigma x_1}{3}, \frac{\Sigma y_1}{3}\right)$

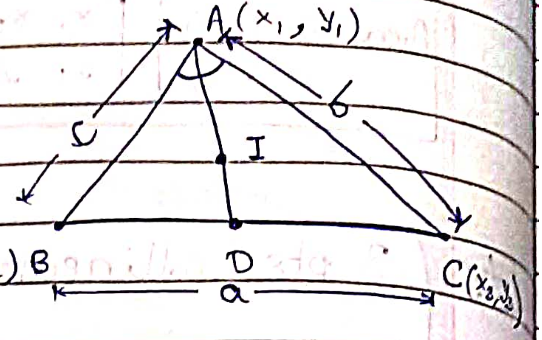


✓ Centroid of ABC $\stackrel{?}{=}$ Centroid of DEF

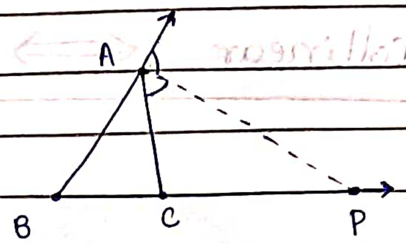
✓ $[ABC] = 4\,[DEF]$

ii) Incentre : $\cap$ of $\angle$ bisectors

✓ $\left(\frac{BD}{DC}\right) = \left(\frac{c}{b}\right)$



✓ $\left(\frac{PB}{PC}\right) = \left(\frac{c}{b}\right)$



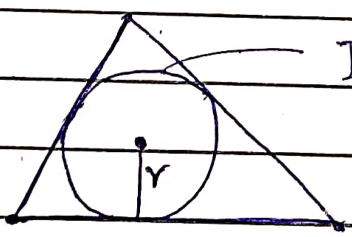
✓ $\left(\frac{AI}{ID}\right) = \left(\frac{b+c}{a}\right)$

✓ $I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

Proof : $D = \left(\frac{bB + cC}{b+c}\right)$ $\left(\text{using } \frac{BD}{DC} = \frac{c}{b}\right)$

$\Rightarrow$ $I = \left(\frac{aA + (b+c)D}{a + (b+c)}\right)$ $\left(\text{using } \frac{AI}{ID} = \frac{b+c}{a}\right)$

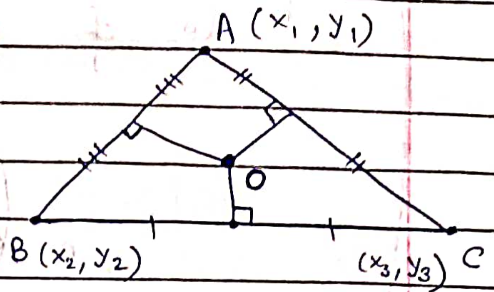
$\Rightarrow$ $I = \left(\frac{aA + bB + cC}{a+b+c}\right)$



Incircle

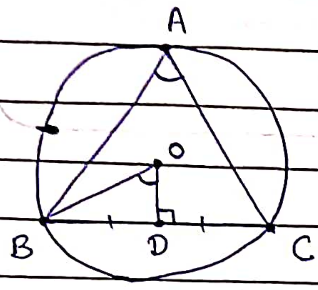
r = Inradius.

iii) Circumcentre : $\cap$ of $\perp$ bisectors.

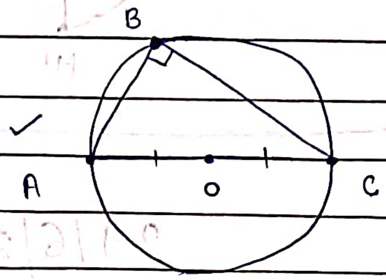


✓ $O = \left(\dfrac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \dfrac{\sum y_1 \sin 2A}{\sum \sin 2A} \right)$

✓



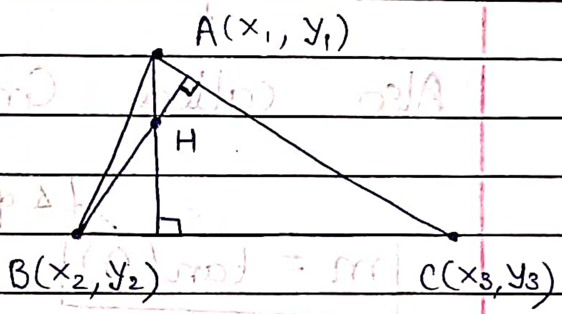
✓



$\angle BAC = \angle BOD$

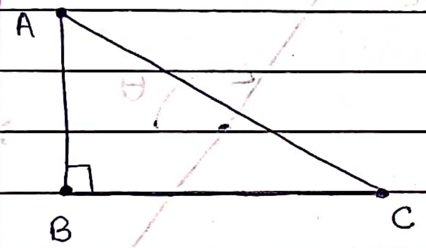
In right ΔABC, $AO = OC$

iv) Orthocentre : $\cap$ of altitudes



✓ $H \equiv \left(\dfrac{\sum x_1 \tan A}{\sum \tan A}, \dfrac{\sum y_1 \tan A}{\sum \tan A} \right)$

✓

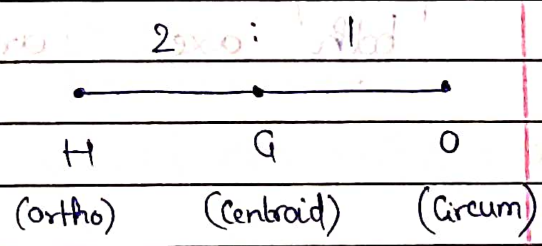


In right angle Δ where $B = 90°$;

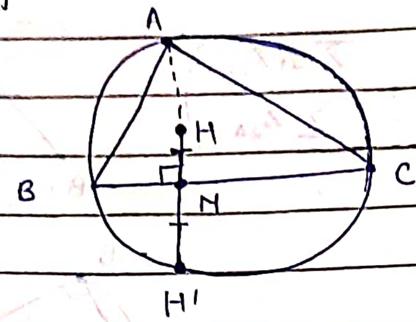
$H \equiv B$

☆ In any triangle,

$HG = 2GO$



☆ Reflection of H (Ortho) about any side of Δ lies on Circumcircle.



$HM = MH'$

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Slope of a Line –

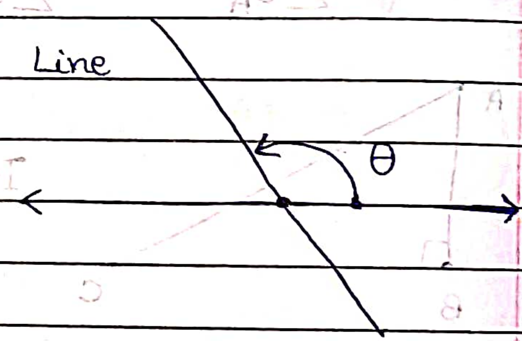
It is tangent of angle made with (+ve) dirxⁿ of X axis.

Also called Gradient. Symbol : m

(Δ of inclination)

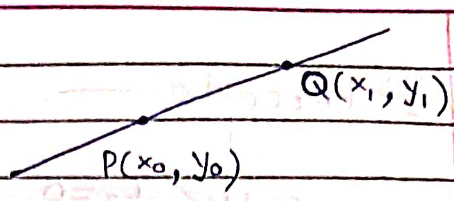
$\boxed{m = \tan(\theta)}$

where $\boxed{0 \leq \theta < \pi}$



✓ Slopes of lines equally inclined to both axes are 1 or −1

$$\boxed{m = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)}$$



☆ If during calc. of slope in a question, highest power of eq^n in slope 'm' gets cancelled, then $\boxed{m = \infty}$ is a sol^n.

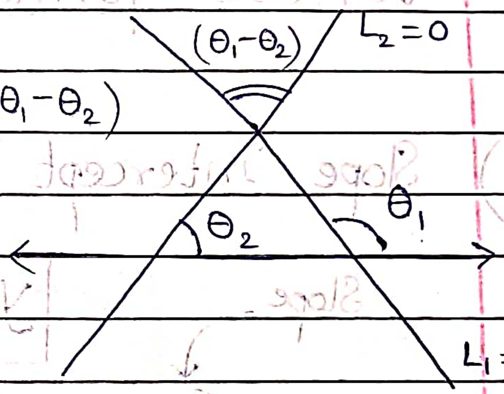
Angle b/w Lines :-

$L_1 = 0$, slope $= m_1$

$L_2 = 0$, slope $= m_2$

$m_1, m_2 \neq \infty$

$\alpha = (\theta_1 - \theta_2)$



$m_1 = t_{\theta_1}$, $m_2 \; t_{\theta_2}$

If α is angle b/w lines then,

$$\boxed{\tan(\alpha) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$$

✓ If lines are $\parallel$ $\Longleftrightarrow$ $\boxed{m_1 = m_2}$

✓ If lines are $\perp$ $\Longleftarrow$ $\boxed{m_1 m_2 = (-1)}$

(Converse not true. Exception : $m_1 = 0$, $m_2 = \infty$)

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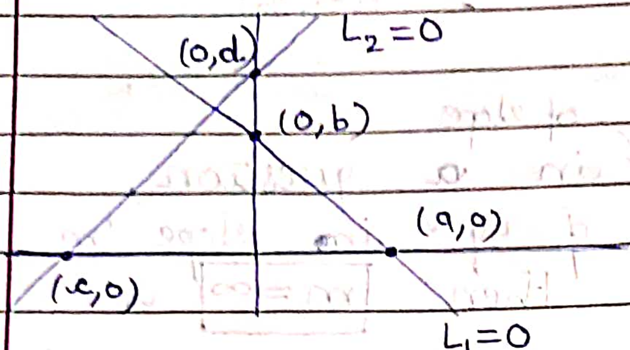
Intercepts —



$L_1=0$

X Intercept : a
Length : |a|

Y Intercept : b
Length : |b|

Various forms of Strt Line

1) Slope Intercept form :

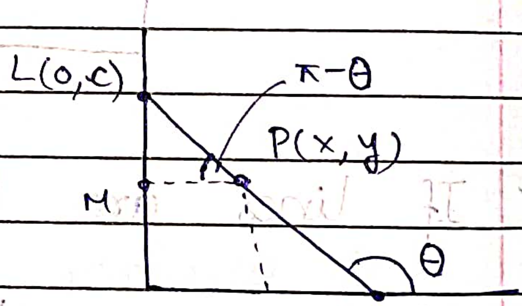
Slope

$$\boxed{y = mx + c}$$

where $m = \tan(\theta)$ & $c = y$ Intercept

Proof :

In ΔPLM,



$$-t_\theta = \left(\frac{LM}{MP}\right)$$

$$\Rightarrow -t_\theta = \left(\frac{c-y}{x}\right)$$

$$\Rightarrow y = t_\theta x + c \Rightarrow y = mx + c$$

2) Point Slope form:

Line passing thru (x_0, y_0) with slope m

$$\boxed{(y-y_0) = m(x-x_0)}$$

Proof: $y_0 = mx_0 + c \Rightarrow c = (y_0 - mx_0)$

$$y = mx + (y_0 - mx_0) \Rightarrow (y-y_0) = m(x-x_0)$$

3) Two Pt. form:

Line thru $A(x_0, y_0)$ & $B(x_1, y_1)$.

$$\boxed{(y-y_0) = \left(\frac{y_1-y_0}{x_1-x_0}\right)(x-x_0)}$$

Proof: $y_0 = mx_0 + c$, $y_1 = mx_1 + c$

$$\Rightarrow (y_1-y_0) = m(x_1-x_0) \Rightarrow m = \left(\frac{y_1-y_0}{x_1-x_0}\right)$$

$$\Rightarrow (y-y_0) = \left(\frac{y_1-y_0}{x_1-x_0}\right)(x-x_0)$$

4) Intercept Form:

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

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where a & b are X & Y Intercepts respectively.

Proof : $(y-0) = \left(\frac{0-b}{a-0}\right)(x-a) \Rightarrow ay + bx = ab$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

5) Normal Form :

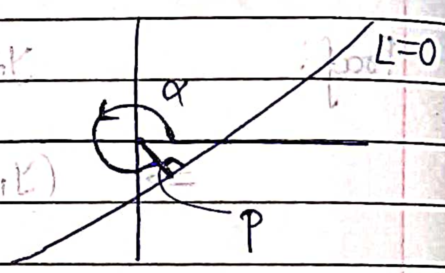
$$\boxed{x\cos(\alpha) + y\sin(\alpha) = p}$$

where α = $\angle$ made by normal drawn from origin with (+ve) dirx^n of X axis.

and p = Length of Normal drawn from origin

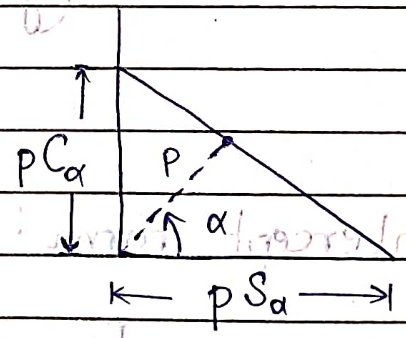
Note :- $0 \leq \alpha < 2\pi$

$p \geq 0$



Proof :

$$\frac{x}{pS_\alpha} + \frac{y}{pC_\alpha} = 1$$



$$\Rightarrow xC_\alpha + yS_\alpha = p$$

6) General Eq^n :

$$\boxed{Ax + By + C = 0} \quad ; \quad A, B, C \in \mathbb{R}$$

Slope Intercept : $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$

Intercept : $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$

Normal form : $\left(\frac{-A}{\sqrt{A^2+B^2}}\right)x + \left(\frac{-B}{\sqrt{A^2+B^2}}\right)y = \left(\frac{C}{\sqrt{A^2+B^2}}\right)$
(if $C > 0$)

Q) For what values of K are pts $(K, 2-2K)$, $(1-K, 2K)$ and $(-4-K, 6-2K)$ are collinear ?

A) $2(\text{Area}) = \begin{vmatrix} 1 & 1 & 1 \\ K & 1-K & -4-K \\ 2-2K & 2K & 6-2K \end{vmatrix}$

$$= \begin{array}{ccc} (+) & (-) & (+) \end{array}$$
$$\begin{vmatrix} 1 & 2 & 0 \\ K & 1 & -4-2K \\ 2-2K & 2 & 4 \end{vmatrix} = \begin{matrix} (4 + 2(4+2K)) \\ -(2)(4K - (2K-2)(2K+4)) \end{matrix}$$

$$= 4 + 8 + 4K - (2)(\cancel{4K} - 4K^2 - \cancel{4K} + 8) = 8K^2 + 4K - 4$$

For collinearity, Area $= 0$

$\Rightarrow \quad 2K^2 + K - 1 = 0 \Rightarrow (2K-1)(K+1) = 0$

$\Rightarrow \quad \boxed{K = 1/2, (-1)}$

reject

bcos at $K = 1/2$, $(K, 2-2K) \equiv (1-K, 2K)$

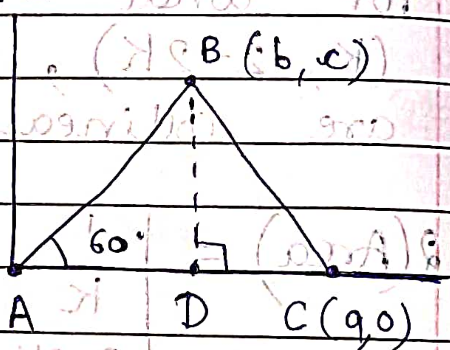
Q) Show that a triangle with one angle as 60°, can NOT have all integral coordinates.

A) Let $A \equiv (0,0)$; $B \equiv (b, c)$; $C \equiv (a, 0)$

where $a \in \mathbb{Z} - \{0\}$ & $b \neq 0$ $c \neq 0$

Let WLOG $\angle CAB = 60^\circ$.

Let $BD \perp AC$ where D is a pt. on AC



In ΔDAB, $\tan(60^\circ) = \left(\frac{BD}{AD}\right) = \left(\frac{c}{b}\right)$

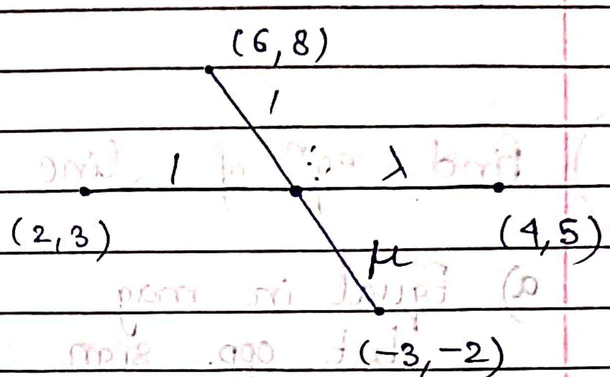
$\Rightarrow \quad c = b\sqrt{3}$

This means at least one of b or $c \notin \mathbb{Z}$

If Δ translated ⇒ At least 1 of b or c Irr. ✓
If Δ rotate about B ⇒ At least 1 of b or c Irr. ✓
Any combination of rotation about B & translation ⇒ ✓

Q) Find ratio in which line by joining (2, 3) and (4, 5) is divided by line joining (6, 8) and (−3, −2).

A) Let it divide in ratio $1 : \lambda$.



By Section formula,

$$\left(\frac{2\lambda+4}{1+\lambda}, \frac{3\lambda+5}{1+\lambda}\right) \equiv \left(\frac{6\mu-3}{\mu+1}, \frac{8\mu-2}{\mu+1}\right)$$

X coordinate : $(2\lambda+4)(\mu+1) = (6\mu-3)(\lambda+1)$

$$\Rightarrow \quad 7 + 5\lambda = 4\mu\lambda + 2\mu$$

$$\Rightarrow \quad \mu = \left(\frac{5\lambda+7}{4\lambda+2}\right)$$

Y coordinate : $\left(\frac{3\lambda+5}{1+\lambda}\right) = \left(\frac{8\mu-2}{\mu+1}\right)$

$$\Rightarrow \left(\frac{3\lambda+5}{1+\lambda}\right) = \frac{8(5\lambda+7)-2(4\lambda+2)}{(5\lambda+7)+(4\lambda+2)} = \left(\frac{32\lambda+52}{9\lambda+9}\right)$$

$$\Rightarrow (\lambda+1)\left[9(3\lambda+5)-(32\lambda+52)\right] = 0$$

$$\Rightarrow (\lambda+1)(5\lambda+7) = 0 \quad \Rightarrow \quad \lambda = -7/5$$

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$\lambda = (-1)$ is impossible.

$\therefore$ It divides in 5 : 7 externally.

Q) Find eqn of line thru (3, 4) and have interce

a) Equal in mag. but opp. sign.   b) with sum 14.

A) a) Let eqn be $\frac{x}{a} + \frac{y}{b} = 1$. So,

$3/a + 4/b = 1$ & $a = -b$

$\Rightarrow$ $3/a - 4/a = 1$ $\Rightarrow$ $a = (-1)$, $b = 1$

$\Rightarrow$ $y = x + 1$

b) Let eqn be $x/a + y/b = 1$. So,

$3/a + 4/b = 1$ & $a + b = 14$

$\Rightarrow$ $3(14-a) + 4a = a(14-a)$ $\Rightarrow$ $a^2 - 13a + 42 = 0$

$\Rightarrow$ $(a-6)(a-7)$ $\Rightarrow$ $(a, b) \equiv (6, 8), (7, 7)$

$\Rightarrow \quad x + y = 7 \quad \& \quad 4x + 3y = 24$

Q) Find eq^n of line on which ⊥ from origin makes 30° with X axis and forms a triangle with area $50/\sqrt{3}$ with axes.

A) Let line be $x\cos\alpha + y\sin\alpha = p$.

We have $\alpha = 30°$ $\Rightarrow x\sqrt{3} + y = 2p$

$\Rightarrow \dfrac{x}{(2p/\sqrt{3})} + \dfrac{y}{(2p)} = 1 \Rightarrow$ Area $= \left(\dfrac{1}{2}\right)\left(\dfrac{2p}{\sqrt{3}}\right)(2p) = \dfrac{50}{\sqrt{3}}$

$\Rightarrow p = 5$

$\Rightarrow \quad x\sqrt{3} + y = 10$

Q) Reduce $x + y\sqrt{3} + 4 = 0$ to –

i) Slope Intercept ii) Intercept iii) Normal

A) i) $y\sqrt{3} = -x - 4 \Rightarrow y = \left(\dfrac{-1}{\sqrt{3}}\right)x + \left(\dfrac{-4}{\sqrt{3}}\right)$

ii) $x + y\sqrt{3} = (-4) \Rightarrow \dfrac{x}{(-4)} + \dfrac{y}{(-4/\sqrt{3})} = 1$

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iii) $x + y\sqrt{3} = (-4) \Rightarrow x\left(\frac{-1}{2}\right) + y\left(\frac{-\sqrt{3}}{2}\right) = 2$

$\Rightarrow x\, c_{\frac{4\pi}{3}} + y\, s_{\frac{4\pi}{3}} = 2$

Q) Find eq^n of $\angle$ bisector of $\angle B$ where vertices of ΔABC are $A(-1,-7)$, $B(5,1)$, $C(1,4)$.

A) $AB = 10$, $BC = 5$, $CA = 5\sqrt{5}$

$\Rightarrow I \equiv \left(\frac{5(-1) + 5\sqrt{5}(5) + 10(1)}{10 + 5 + 5\sqrt{5}}, \frac{5(-7) + 5\sqrt{5}(1) + 10(4)}{10 + 5 + 5\sqrt{5}} \right)$

$\Rightarrow I \equiv \left(\frac{1 + 5\sqrt{5}}{3 + \sqrt{5}}, \frac{1 + \sqrt{5}}{3 + \sqrt{5}} \right)$

$\text{Slope}(BI) = \frac{(3+\sqrt{5}) - (1+\sqrt{5})}{5(3+\sqrt{5}) - (1+5\sqrt{5})} = 1/7$

$\Rightarrow BI \equiv 7(y-1) - (x-5)$

Q) 2 equal sides of an isoceles Δ are given by eq^n $7x - y + 3 = 0$ and $x - y = 3$.
Its 3rd side passes thru $(1, -10)$.
Determine eq^n of 3rd side.

A) Let $L_1 \equiv 7x - y + 3$; $L_2 \equiv x - y - 3$

Since it is isoceles Δ, $(L_3 \wedge L_2) = (L_3 \wedge L_1)$

$$\Rightarrow \left|\frac{m_3 - m_2}{1 + m_3 m_2}\right| = \left|\frac{m_3 - m_1}{1 + m_1 m_3}\right|$$

$$\Rightarrow \left|\frac{m-1}{1+m}\right| = \left|\frac{m-7}{1+7m}\right|$$

$$\Rightarrow |(m-1)(1+7m)| = |(m-7)(m+1)|$$

$$\Rightarrow (7m^2 - 6m - 1)^2 = (m^2 - 6m - 7)^2$$

$$\Rightarrow (8m^2 - 12m - 8)(6m^2 + 6) = 0$$

$$\Rightarrow 2m - 3m - 2 = 0 \Rightarrow (2m+1)(m-2) = 0$$

$$\Rightarrow m = 2 \text{ or } -1/2$$

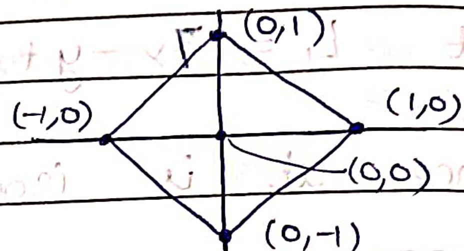
$$\Rightarrow (y+10) = 2(x-1) \quad \text{or} \quad 2(y+10) + (x-1) = 0$$

Q) Find area within following curves -

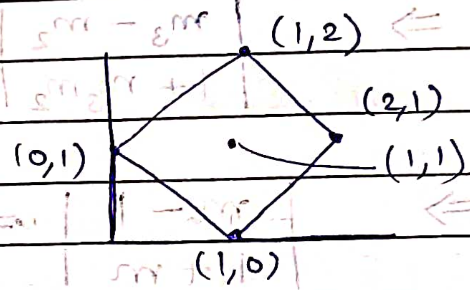
i) $|x|+|y|=1$ ii) $|x-1|+|y-1|=1$

iii) $|x-1|+|y-2|=1$

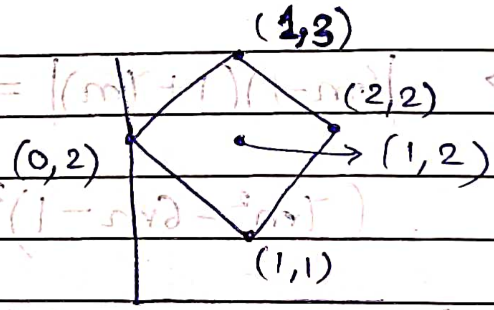
A) i) Area $= 2$



ii) Area $= 2$



iii) Area $= 2$



Q) Find integral values of 'm' for which x coordinate of $\cap$ of lines $3x+4y=9$ & $y=mx+1$ is also an integer.

A) Eliminating y, $(9-3x) = 4(mx+1)$

$$\Rightarrow x = \left(\frac{5}{4m+3}\right)$$

$x \in \mathbb{Z} \Rightarrow (4m+3) | 5 \Rightarrow 4m+3 \in \{-5, -1, +1, +5\}$

$\Rightarrow \boxed{m \in \{-2, -1, -1/2, 1/2\}}$

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Parametric form of Strt. Line –

Eqn of strt. line passing thru a fixed pt. (x_0, y_0) and making angle θ with (+ve) dirxn of X axis.

[Para. generally gives better soln]

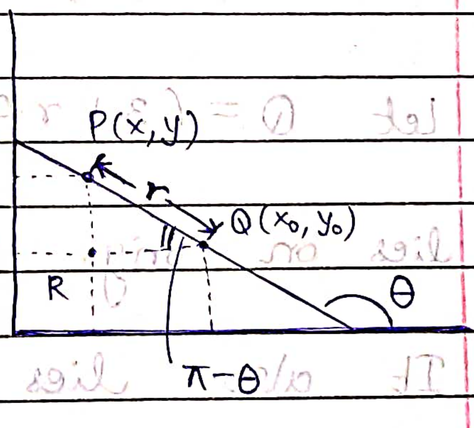
$$\boxed{\left(\frac{x - x_0}{c_\theta}\right) = \left(\frac{y - y_0}{s_\theta}\right) = r}$$

☆ $x = x_0 + r c_\theta$
$y = y_0 + r s_\theta$
(More Useful)

Here, r = dist. of any pt. $P(x, y)$ from a fixed pt. $Q(x_0, y_0)$.

It is also called 'parameter'.

Proof: In ΔPQR,



$c_{\pi-\theta} = (-c_\theta) = \left(\frac{x_0 - x}{r}\right)$

$s_{\pi-\theta} = s_\theta = \left(\frac{y - y_0}{r}\right)$

$\Rightarrow \boxed{r = \left(\frac{x - x_0}{c_\theta}\right) = \left(\frac{y - y_0}{s_\theta}\right)}$

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Eg - $x + y = 2 \Rightarrow t_\theta = (-1)$

$\Rightarrow c_\theta = (-1/\sqrt{2})$, $s_\theta = 1/\sqrt{2}$

Since line pases thru (1, 1); we get

$$\left(\frac{x-1}{-1/\sqrt{2}}\right) = \left(\frac{y-1}{1/\sqrt{2}}\right) = r$$

☆ If we have to consider any pt. on a given line, in parametric form we ALWAYS consider

$$(x_0 + r c_\theta, \; y_0 + r s_\theta)$$

☆ Q) If a strt. line thru P(3, 4) makes angle $\pi/6$ with (+ve) dirx^n of X axis, and meets line $12x + 5y + 10 = 0$ at Q. find PQ.

A) Let $Q = (3 + r c_{\pi/6}, \; 4 + r s_{\pi/6})$ since it lies on orig. strt. line.

It also lies on $12x + 5y + 10 = 0$

$\Rightarrow 12(3 + r\cos\pi/6) + 5(4 + r\sin\pi/6) + 10 = 0$

$\Rightarrow 66 + 6\sqrt{3}r + \left(\frac{5r}{2}\right) = 0 \Rightarrow \boxed{r = -\left(\frac{132}{5+12\sqrt{3}}\right)}$

$\Rightarrow \boxed{PQ = \left(\frac{132}{5+12\sqrt{3}}\right)}$

Q) Find dirx^n in which a strt. line must be drawn thru (1,2) s.t. its ∩ with $x+y=4$ may be at a dist $\sqrt{6}/3$ from this pt.

A) Let pt. be $\left(1 + \frac{\sqrt{6}}{3}c_\theta, 2 + \frac{\sqrt{6}}{3}s_\theta\right)$.

This lies on $x+y=4$.

$\Rightarrow \left(1 + \frac{\sqrt{6}}{3}c_\theta\right) + \left(2 + \frac{\sqrt{6}}{3}s_\theta\right) = 4$

$\Rightarrow s_\theta + c_\theta = \sqrt{3/2} \Rightarrow s_{\theta+\pi/4} = \frac{\sqrt{3}}{2}$

$\Rightarrow \boxed{\theta = 15', 75'}$

Q) A line thru (2, 3) makes $3\pi/4$ angle with (−ve) dirxⁿ of X axis. Find length of line segment cutoff b/w (2,3) and $x+y=7$

A) $3\pi/4 \angle$ with (−ve) X axis $\longleftrightarrow$ $\pi/4$ with (+ve) X axis.

Let P be $(2 + r\cos_{\pi/4}, \; 3 + r\sin_{\pi/4})$

Since it lies on $x+y=7$

$$\Rightarrow (2 + r\cos_{\pi/4}) + (3 + r\sin_{\pi/4}) = 7$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$

Q) Find dist. of (2, 3) from $2x - 3y + 9 = 0$ measured along $2x - 2y + 5 = 0$

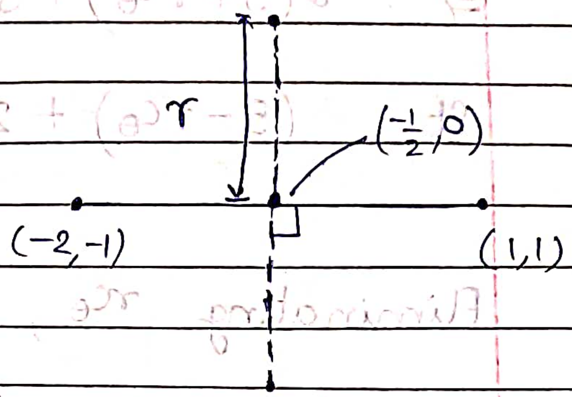
A) In this case $\theta = \pi/4$. Let pt. of line be $\left(2 + r\cos_{\frac{\pi}{4}}, \; 3 + r\sin_{\frac{\pi}{4}}\right)$.

$$\Rightarrow 2\left(2 + \frac{r}{\sqrt{2}}\right) - 3\left(3 + \frac{r}{\sqrt{2}}\right) + 9 = 0 \Rightarrow \boxed{r = 4\sqrt{2}}$$

Q) The extremeties of diag. of sq. are (1,1) and (−2,−1). Obtain the other 2 vertices, and eqⁿ of other diag.

A) We have,

$\tan(\theta)\left(\dfrac{1+1}{1+2}\right) = (-1)$



& $2r = \sqrt{2^2+3^2}$

$\Rightarrow \quad t_\theta = (-3/2) \quad \& \quad r = \sqrt{13}/2$

$\Rightarrow \quad x = (-1/2) \pm (\sqrt{13}/2)(-2/\sqrt{13})$
$\quad\; y = 0 \pm (\sqrt{13}/2)(3/\sqrt{13})$

$\Rightarrow$ Pts. $\equiv (-3/2, 3/2) \; ; \; (1/2, -3/2)$

$\Rightarrow$ Diag $\equiv \quad 2y + 3x + 3/2 = 0$

Q) The sides AB and AC of a ΔABC are respectively $2x+3y=29$ & $x+2y=16$. If midpt. of BC is (5,6), find BC eqⁿ.

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A) Let pts $B, C \equiv (5 \pm rc_\theta, 6 \pm rs_\theta)$

$\Rightarrow \quad 2(5+rc_\theta) + 3(6+rs_\theta) = 29$

& $\quad (5-rc_\theta) + 2(6-rs_\theta) = 16$

Eliminating rc_θ,

$1 + 3rs_\theta - 4rs_\theta = 0 \Rightarrow rs_\theta = 1$

$\Rightarrow rc_\theta = (-1)$

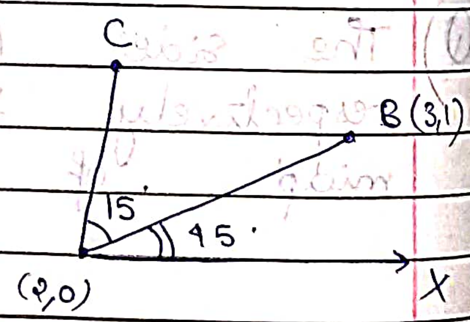
$\Rightarrow \quad r = \sqrt{2}, \quad t_\theta = (-1)$

$\Rightarrow \quad BC \equiv (y-6) = -(x-5)$

$\Rightarrow \quad BC \equiv \boxed{x+y-11=0}$

Q) Line joining $A(2,0)$ and $B(3,1)$ is rotated about A in ↺ thru $15°$. Find eq^n of line in new post.
If B goes to C in new post., find C.

A) $m = \left(\frac{1-0}{3-2}\right) = 1 \Rightarrow \theta = 45°$



Hence for line AC, $t\phi = t_{60^\circ} = \sqrt{3}$

$\Rightarrow$ $\boxed{AC \equiv y - x\sqrt{3} + 2\sqrt{3} = 0}$

Now, $AB = \sqrt{2} \Rightarrow x_C = 2 + \sqrt{2}\, c_{60^\circ}$
$y_C = 0 + \sqrt{2}\, s_{60^\circ}$

$\Rightarrow$ $\boxed{C \equiv \left(2 + 1/\sqrt{2},\ \sqrt{3/2}\right)}$

Q) If $y - \sqrt{3}x + 3 = 0$ cuts $y^2 = x + 2$ at A & B, find PA · PB where $P \equiv (\sqrt{3}, 0)$.

A) Observe that P lies on given line.

Let A, B $\equiv (\sqrt{3} + r c_{60^\circ},\ 0 + r s_{60^\circ})$ where $r = r_A, r_B$

$\Rightarrow (r s_{60^\circ})^2 = (\sqrt{3} + r c_{60^\circ}) + 2$

$\Rightarrow \left(\frac{3r^2}{4}\right) = (2 + \sqrt{3}) + \frac{r}{2}$

$\Rightarrow 3r^2 - 2r + 4(2 + \sqrt{3}) = 0 \begin{array}{l} r_A \\ r_B \end{array}$

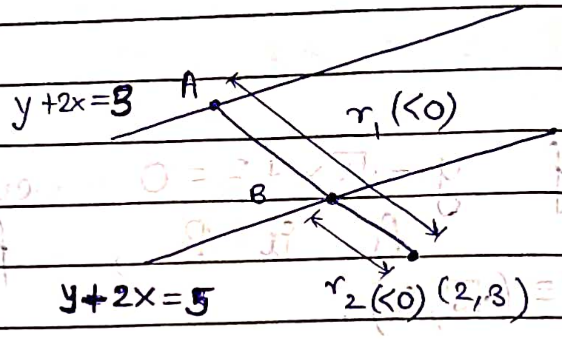
$\Rightarrow |r_A r_B| = PA \cdot PB = \left(\frac{4}{3}\right)(2 + \sqrt{3})$

☆ Q) Find eqⁿ of line thru (2, 3) and making intercept of length 2 units b/w $y+2x=3$ and $y+2x=5$

A) Let $A \equiv (2+r_1 c_\theta,\ 3+r_1 s_\theta)$ on $y+2x=3$
and $B \equiv (2+r_2 c_\theta,\ 3+r_2 s_\theta)$ on $y+2x=5$

We get,
$$4 + r_1(s_\theta + 2c_\theta) = 0$$
$$2 + r_2(s_\theta + 2c_\theta) = 0$$

$\Rightarrow (r_1 - r_2)(s_\theta + 2c_\theta) = (-2)$



$\Rightarrow s_\theta + 2c_\theta = (\mp 1)$

$\Rightarrow 4 - 4s_\theta^2 = s_\theta^2 \mp 2s_\theta + 1$

$\Rightarrow 5s_\theta^2 \mp 2s_\theta - 3 = 0 \Rightarrow (5s_\theta \pm 3)(s_\theta \mp 1) = 0$

$\Rightarrow s_\theta = (+3/5),\ \pm 1$ — Impossible
$c_\theta = (\mp 4/5),\ 0$

$t_\theta = -3/4$ $\qquad$ $\theta = \pi/2$

$\Rightarrow \boxed{(y-3) = (-3/4)(x-2)}$

$\boxed{x = 2}$

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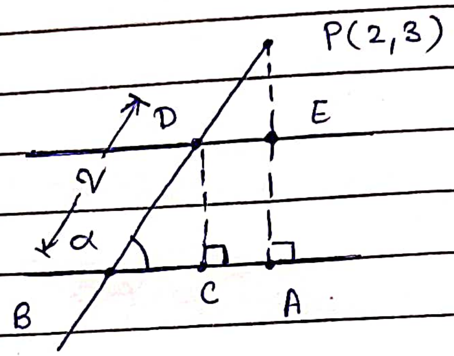
★ Q) find eq^n of line thru (2,3) and making intercept of length 2 units b/w $y+2x=3$ and $y+2x=5$

A) $AE = DC = \left|\dfrac{5-3}{\sqrt{1^2+2^2}}\right|$



$\Rightarrow \quad DC = 2/\sqrt{5}$

Now, $DB\sin(\alpha) = DC \Rightarrow \tan(\alpha) = \left(\dfrac{DC}{BC}\right)$

$\Rightarrow \quad \left|\dfrac{m+2}{1-2m}\right| = \left(\dfrac{2/\sqrt{5}}{4/\sqrt{5}}\right) = \left(\dfrac{1}{2}\right)$

$\Rightarrow \quad 4(m+2)^2 = (2m-1)^2$

$\Rightarrow \quad 4m^2 + 16m + 16 = 4m^2 - 4m + 1$

$m = \infty$ $\qquad$ $m = (-3/4)$

$\Rightarrow \quad \boxed{x=2} \; ; \; \boxed{4y + 3x - 18 = 0}$

Q) A line thru $A(-5,-4)$ meets $x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at B, C, D respectively. If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$, find eqn of line.

A) Let $B \equiv (-5 + r_1 c_\theta, -4 + r_1 s_\theta)$. It lies on $x+3y+2=0$.

$\Rightarrow \quad (r_1 c_\theta - 5) + 3(r_1 s_\theta - 4) + 2 = 0$

$\Rightarrow \quad r_1 = \left(\frac{15}{c_\theta + 3s_\theta}\right)$

Similarly, $r_2 = \left(\frac{10}{2c_\theta + s_\theta}\right)$, $r_3 = \left(\frac{6}{c_\theta - s_\theta}\right)$

$\left(\frac{15}{r_1}\right)^2 + \left(\frac{10}{r_2}\right)^2 = \left(\frac{6}{r_3}\right)^2 \Rightarrow (c_\theta + 3s_\theta)^2 + (2c_\theta + s_\theta)^2 = (c_\theta - s_\theta)^2$

$\Rightarrow 5c_\theta^2 + 10s_\theta^2 + 10s_\theta c_\theta = c_\theta^2 + s_\theta^2 - 2s_\theta c_\theta$

$\Rightarrow 4c_\theta^2 + 9s_\theta^2 + 12s_\theta c_\theta = 0$

$\Rightarrow (2c_\theta + 3s_\theta)^2 = 0 \Rightarrow t_\theta = (-2/3)$

$\Rightarrow \boxed{3y + 2x + 22 = 0}$

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Imp. Results –

1) Eqⁿ of line $\parallel$ to $ax + by + c = 0$ is

$$ax + by + \lambda = 0$$

2) Eqⁿ of line $\perp$ to $ax + by + c = 0$ is

$$bx - ay + \lambda = 0$$

3) Length of $\perp$ from $P(\alpha, \beta)$ to line $ax + by + c = 0$ is

$$\left| \frac{a\alpha + b\beta + c}{\sqrt{a^2+b^2}} \right|$$

Proof: Let $PQ \perp$ line s.t. Q on line.

$\Rightarrow \quad Q \equiv (\alpha + r c_\theta, \beta + r s_\theta)$ where $t_\theta = b/a$

$$\Rightarrow \quad a\left(\alpha + \frac{r \cdot a}{\sqrt{a^2+b^2}}\right) + b\left(\beta + \frac{r \cdot b}{\sqrt{a^2+b^2}}\right) + c = 0$$

$$\Rightarrow \quad \left(\frac{a\alpha\sqrt{a^2+b^2} + b\beta\sqrt{a^2+b^2} + a^2 r + b^2 r + c\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}\right) = 0$$

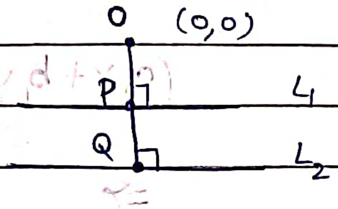
$$\Rightarrow \quad r = -\left(\frac{a\alpha + b\beta + c}{\sqrt{a^2+b^2}}\right) \Rightarrow \boxed{|r| = \left|\frac{a\alpha + b\beta + c}{\sqrt{a^2+b^2}}\right|}$$

4) Dist. b/w || lines $ax+by+c_1=0$ & $ax+by+c_2=0$ is

$$\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

Proof:

Dist. $= |OP - OQ|$



5) Dist. b/w $y=mx+c_1$ & $y=m_2x+c_2$ is

$$\left|\frac{c_1-c_2}{\sqrt{1+m^2}}\right|$$

6) Pt. of $\cap$ of Lines

7) Concurrent Lines — $L_1=0$, $L_2=0$, $L_3=0$

$L_i \equiv a_i x + b_i y + c_i$; $i=1,2,3$

for concurrency, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

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Family of Lines —

Family of lines thru the • of ∩ of lines $L_1 : a_1x + b_1y + c_1 = 0$ & $L_2 : a_2x + b_2y + c_2 = 0$ is given by

$$(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0 \; ; \; \lambda \in \mathbb{R}$$

$$\Rightarrow \quad L_1 + \lambda L_2 = 0 \quad ; \quad \lambda \in \mathbb{R}$$

Conversely, the eqⁿ $L_1 + \lambda L_2 = 0$ represents family of lines passing thru • of ∩ of $L_1 = 0$ & $L_2 = 0$

Q1) Find line passing thru $(2,1)$ & thru pt. of ∩ of $x + 2y = 3$ & $2x - 3y = 4$.

A) Let line be $(x + 2y - 3) + \lambda(2x - 3y - 4) = 0$

Since it passes thru $(2,1)$,

$$(2 + 2 - 3) + \lambda(4 - 3 - 4) = 0 \Rightarrow \lambda = 1/3$$

$$\Rightarrow \quad \boxed{5x + 3y - 13 = 0}$$

Q) If $3a + 2b + 6c = 0$, then pt. family of lines $ax + by + c = 0$ passes thru a fixed pt. find the pt.

A) $c = -\left(\frac{3a+2b}{6}\right) \Rightarrow L \equiv ax + by - \left(\frac{3a+2b}{6}\right) = 0$

$$\Rightarrow (3a)(2x-1) + (2b)(3y-1) = 0$$

This is a family of lines with members

$(2x-1) = 0$
& $(3y-1) = 0$ $\Rightarrow$ $x = 1/2$, $y = 1/3$ for all members $\Rightarrow$ Fixed Pt.

Q) If $4a^2 + 9b^2 - c^2 + 12ab = 0$ then $ax + by + c = 0$ passes thru fix. pt. Find the pt.

A) $(4a^2 + 9b^2 + 12ab) - c^2 = 0 \Rightarrow 2a + 3b + c = 0$
$\Rightarrow 2a + 3b - c = 0$

$\Rightarrow$ Family 1 : $(2, 3)$; Family 2 : $(-2, -3)$

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Q) Find lines passing thru $\cap$ of $4x-3y-1=0$ & $2x-5y+3=0$, and are equally inclined with axes.

A) Let line be $(4x-3y-1)+\lambda(2x-5y+3)=0$

$\Rightarrow (2\lambda+4)x-(5\lambda+3)y+(3\lambda-1)=0$

For equally inclined to axes $m=1,-1$

$\Rightarrow \left(\dfrac{2\lambda+4}{5\lambda+3}\right)=1,-1$

$\Rightarrow \lambda = 1/3, (-1)$

$\Rightarrow$ Lines: $14x-14y-3+3=0 \Rightarrow \boxed{x=y}$
$2x+2y-1-3=0 \Rightarrow \boxed{x+y=2}$

Imp. Result

☆Q) If algebraic sum of $\perp$ dist. from n given pts on a variable strt. line is zero, then pt. variable strt. lines passes thru a fix. pt.

A) Let pts. be $(x_1, y_1), \ldots, (x_n, y_n)$.

We are given a variable line $ax + by + c = 0$.

Now algebraic sum,

$$\sum\left(\frac{ax_1 + by_1 + c_1}{\sqrt{a^2+b^2}}\right) = 0 \Rightarrow a(\Sigma x_1) + b(\Sigma y_1) + nc = 0.$$

$$\Rightarrow a\left(\frac{\Sigma x_1}{n}\right) + b\left(\frac{\Sigma y_1}{n}\right) + c = 0 \Rightarrow \boxed{\text{Pt.} \equiv \left(\frac{\Sigma x_1}{n}, \frac{\Sigma y_1}{n}\right)}$$

Q) Find values of non (-ve) real nos. $h_1, h_2, h_3, k_1, k_2, k_3$ s.t. the algebraic sum of $\perp$ from pts. $(2, k_1)$; $(3, k_2)$; $(7, k_3)$; $(h_1, 4)$; $(h_2, 5)$; $(h_3, -3)$ on a variable lines passing thru $(2,1)$ is ~~at~~ zero.

A) By earlier Qs result,

$$\left(\frac{2+3+7+\Sigma h_1}{6}\right) = 2 \quad ; \quad \left(\frac{\Sigma k_1 + 4 + 5 - 3}{6}\right) = 1$$

$$\Rightarrow \Sigma h_i = 0 \quad ; \quad \Sigma k_i = 0$$

Since; $h_i, k_i \geq 0 \Rightarrow \boxed{h_i = k_i = 0 \ ; \ \forall i}$

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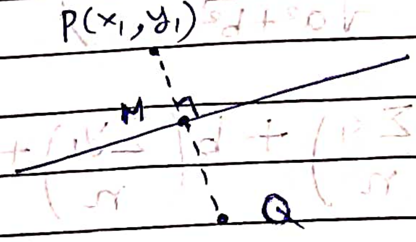
Image & Foot of ⊥ —

Image of pt. $P(x_1, y_1)$ in $ax+by+c=0$



Basic Method:

To find Q,

✓ $\left(\frac{Q+P}{2}\right)$ lies on line

✓ m_{PQ} ⊥ to line.

PQ

Q found ⇒ M found

Results —

C-1 : If (x_2, y_2) image of (x_1, y_1) about $ax+by+c=0$, then

$$\boxed{\left(\frac{x_2-x_1}{a}\right) = \left(\frac{y_2-y_1}{b}\right) = (-2)\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)}$$

C-2 : If (x_2, y_2) is foot of ⊥ from (x_1, y_1) on line $ax+by+c=0$, then

$$\boxed{\left(\frac{x_2-x_1}{a}\right) = \left(\frac{y_2-y_1}{b}\right) = -\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)}$$

Q) Find image of (2,3) about $3x-4y+5=0$. Also find foot of $\perp$ drawn from (2,3).

A) Image :

$$\left(\frac{x-2}{3}\right)=\left(\frac{y-3}{-4}\right)=(-2)\left(\frac{6-12+5}{25}\right)$$

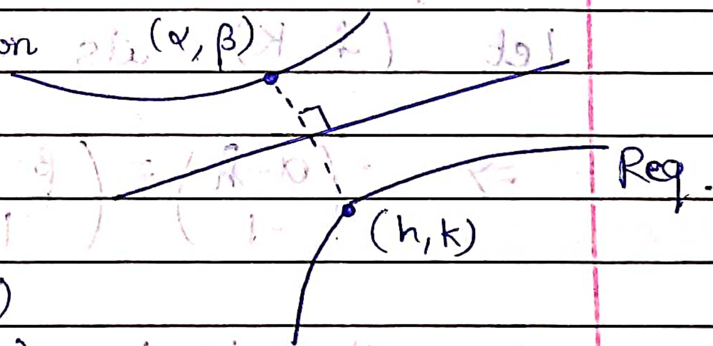
$$\Rightarrow \boxed{x=\left(\frac{56}{25}\right)} ; \boxed{y=\left(\frac{67}{25}\right)}$$

foot of $\perp$:

$$\left(\frac{x-2}{3}\right)=\left(\frac{y-3}{-4}\right)=-\left(\frac{6-12+5}{25}\right)$$

$$\Rightarrow \boxed{x=\left(\frac{53}{25}\right)} ; \boxed{y=\left(\frac{71}{25}\right)}$$

☆ To find image of curve in a Line.



Let (α,β) be a pt. on KNOWN curve.

Assume (h,k) be its image. $\Rightarrow$ $\alpha = f(h,k)$
$\beta = g(h,k)$

Input in KNOWN curve eq^n $\Rightarrow$ Req. Curve found!

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Q) Find image of $y = x^2+1$ in line $y = x$

A) Let (α, β) a pt. on $y = x^2+1$.

Let (h, k) its image.

$$\Rightarrow \left(\frac{\alpha - h}{-1}\right) = \left(\frac{\beta - k}{1}\right) = (-2)\left(\frac{\overset{k}{\beta} - \overset{h}{\alpha}}{2}\right)$$

$$\Rightarrow \alpha = k, \quad \beta = h$$

$$\Rightarrow h = k^2+1 \Rightarrow \text{Req: } \boxed{x = y^2+1}$$

Q) Find image of $y = x^2+1$ in line $y - x = (-1)$

A) Let (α, β) a pt. on $y = x^2+1$

Let (h, k) its image.

$$\Rightarrow \left(\frac{\alpha - h}{-1}\right) = \left(\frac{\beta - k}{1}\right) = (-2)\left(\frac{k - h + 1}{2}\right)$$

$$\Rightarrow \alpha = k+1 \quad ; \quad \beta = h - 1$$

$$\Rightarrow (h-1) = (k+1)^2 + 1 \Rightarrow \text{Req: } \boxed{x = y^2 + 2y + 3}$$

Locus —

Locus is eq^n of path travelled by a moving pt. under ⓐ given geometric conditions.

Let moving pt. be (h, k)

↓ Apply Geometric Condition

Eq^n Parameter INDEPENDENT	Eq^n DEPENDENT on Parameters
↓	↓ Eliminate parameters
$(h,k) \rightarrow (x,y)$	$(h,k) \rightarrow (x,y)$

Q) If $P \equiv (1,0)$; $Q \equiv (-1,0)$; $R \equiv (2,0)$, find locus of pt. S under the condition
$$SQ^2 + SR^2 = 2 \cdot SP^2.$$

Q) For variable ΔABC, $C(1,2)$, $A(c_t, s_t)$, $B(s_t, -c_t)$ resp. Find locus of centroid of ΔABC.

Q) A variable line drawn thru $(1,3)$ meets X axis at A & Y axis at B. If rectangle OABP is $O(0,0)$ is completed, then find locus of P.

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A) Let $S \equiv (x, y)$.

$$\Rightarrow \left((x+1)^2+y^2\right)+\left((x-2)^2+y^2\right)=2\left((x-1)^2+y^2\right)$$

$$\Rightarrow (x^2+2x+1)+(x^2-4x+4)=2(x^2-2x+1)$$

$$\Rightarrow \boxed{2x+3=0}$$

A) $$G \equiv \left(\frac{1+c_t+s_t}{3}, \frac{2+s_t-c_t}{3}\right) \equiv (x, y)$$

$$\Rightarrow s_t+c_t=(3x-1) \quad ; \quad s_t-c_t=(3y-2)$$

$$\Rightarrow \boxed{(3x-1)^2+(3y-2)^2=2}$$

A) Let $P \equiv (h, k) \Rightarrow A \equiv (h, 0)$; $B \equiv (0, k)$

$$\Rightarrow \text{Line} \equiv \left(\frac{x}{h}\right)+\left(\frac{y}{k}\right)=1$$

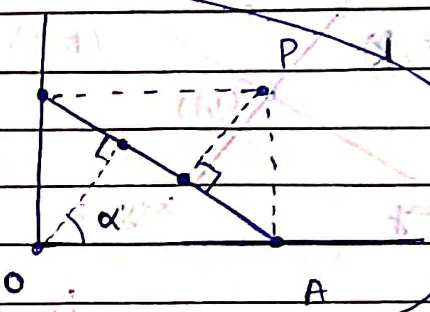
Since it passes thru $(1, 3)$;

$$\Rightarrow \left(\frac{1}{h}\right)+\left(\frac{3}{k}\right)=1 \Rightarrow \text{Locus} \equiv \boxed{y+3x=xy}$$

Q) The ends A & B of a line segment of const. length c, slide on fix. rect. axes OX, OY resp. If rect. OAPB is completed then show that the locus of foot of $\perp$ drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

A) By symmetry,

alt. from O = alt. from P



Let $A(a, 0) \Rightarrow B(0, \sqrt{c^2-a^2}) \Rightarrow P \equiv (a, \sqrt{c^2-a^2})$

Now line $AB \equiv x\sqrt{c^2-a^2} + ya - a\sqrt{c^2-a^2} = 0$

Let foot of $\perp$ be (h, k)

$\Rightarrow \left(\dfrac{h-a}{\sqrt{c^2-a^2}}\right) = \left(\dfrac{k-\sqrt{c^2-a^2}}{a}\right) = (-1)\left(\dfrac{a\sqrt{c^2-a^2}}{c^2}\right)$

$\Rightarrow h = a^3/c^2, \quad k = (c^2-a^2)^{3/2}/c^2$

Now, $h^{2/3} + k^{2/3} = a^2/c^{4/3} + (c^2-a^2)/c^{4/3} = c^{2/3}$

$\Rightarrow$ Locus : $\boxed{x^{2/3} + y^{2/3} = c^{2/3}}$

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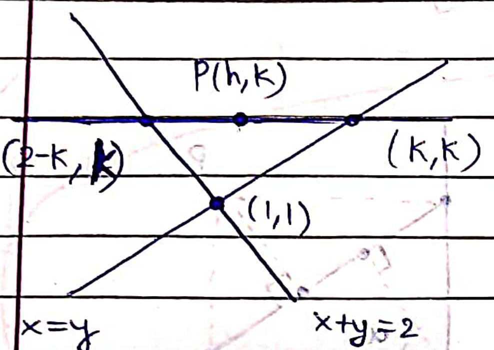
Q) Area of Δ formed by $\cap$ of a line $\parallel$ to X axis and passing thru P(h, K) with lines $y=x$ and $x+y=2$ is $4h^2$. Find locus of pt. P.

A) 

Area $= 4h^2$

$$\Rightarrow \left|\begin{vmatrix} 1 & 1 & 1 \\ 1 & K & 2-K \\ 1 & K & K \end{vmatrix}\right| = 8h^2$$

$$\Rightarrow \left|\begin{vmatrix} 1 & 0 & 0 \\ 1 & k-1 & 1-K \\ 1 & k-1 & K-1 \end{vmatrix}\right| = 8h^2$$

$$\Rightarrow \left|(K-1)^2 + (K-1)^2\right| = 8h^2$$

$$\Rightarrow (K-1)^2 = 4h^2$$

$\Rightarrow$ Locus : $\boxed{\begin{array}{l} y+2x-1=0 \\ y-2x-1=0 \end{array}}$

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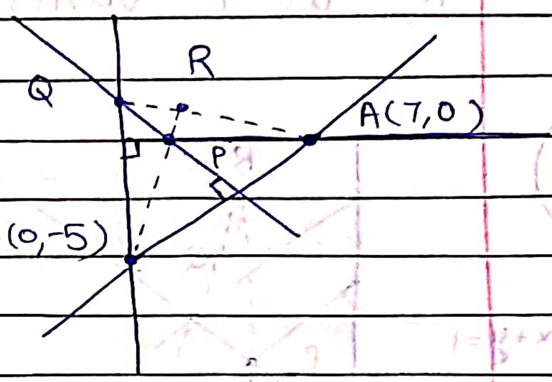
Q) A line cuts X axis at A(7,0) and Y axis at B(0, -5). A variable line PQ is drawn ⊥ to AB cutting X axis in P and Y axis in Q. If AQ and BP intersect at R, find locus of R.

A) Let PQ be



$$5y + 7x + \lambda = 0$$

$$\Rightarrow P\left(\frac{-\lambda}{7}, 0\right)$$

$$Q\left(0, \frac{-\lambda}{5}\right)$$

$$AQ \equiv y = \left(\frac{\lambda}{35}\right)(x-7)$$

$$BP \equiv (y+5) = \left(\frac{-35}{\lambda}\right)(x)$$

Since $R(h,k)$ lies on both,

$$k \cdot (k+5) = (h-7)\cdot(-h)$$

$$\Rightarrow k(k+5) + h(h-7) = 0$$

$$\Rightarrow \text{Locus}: \boxed{x^2 - 7x + y^2 + 5y = 0}$$

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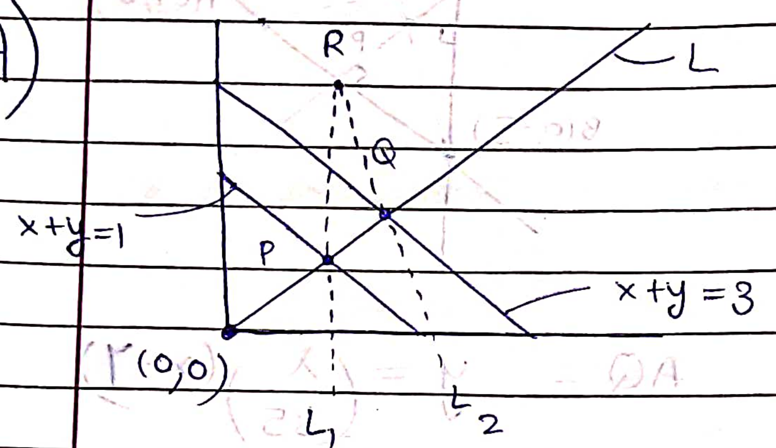
Q) A strt line L thru (0,0) meets the lines $x+y=1$ and $x+y=3$ at P & Q respectively. Thru P & Q, 2 strt. lines L_1 & L_2 are drawn $\parallel$ to $2x-y=5$ and $3x+y=5$ resp. Lines L_1 & L_2 $\cap$ at R. Show that the locus of R, as L varies, is a strt. line.

A)



Let $L: y = mx$

$\Rightarrow P\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$,

$Q\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$

Let $R(h,k)$, we know $m_{RP} = m_{L_1} = 2$

& $m_{RQ} = m_{L_2} = (-3)$

$$\Rightarrow \left(\frac{k(m+1)-m}{h(m+1)-1}\right) = 2 \quad \& \quad \left(\frac{k(m+1)-3m}{h(m+1)-3}\right) = (-3)$$

$$\Rightarrow \boxed{m = \left(\frac{k-2h+2}{k-2h-1}\right)} \quad \& \quad \boxed{m = \left(\frac{k+3h-9}{k+3h-3}\right)}$$

$$\Rightarrow \left(\frac{(k-2h)+2}{(k-2h)-1}\right) = \left(\frac{(k+3h)-9}{(k+3h)-3}\right)$$

$$\Rightarrow \left(\frac{(2k-4h+1)+3}{(2k-4h+1)-3}\right) = \left(\frac{(k+3h-6)-3}{(k+3h-6)+3}\right)$$

$$\Rightarrow \left(\frac{2K-4h+1}{3}\right)+\left(\frac{K+3h-6}{3}\right)=0$$

$$\Rightarrow 3K-h-5=0 \Rightarrow \text{Locus}: \boxed{3y-x-5=0}$$

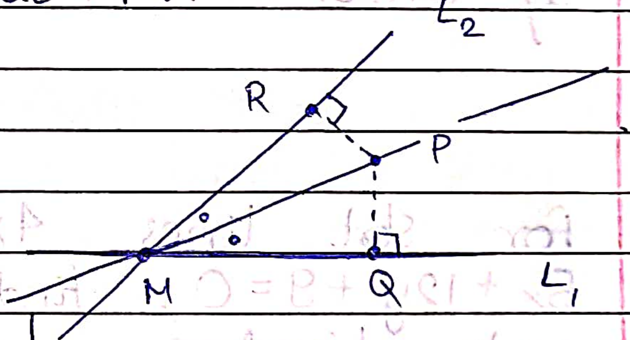
Q Angle Bisector —

Locus of a pt. which moves s.t. the $\perp$ drawn from it to the 2 given $\cap$ lines are equal.

Consider $L_1 : a_1x+b_1y+c_1=0$ & $L_2 : a_2x+b_2y+c_2=0$.

Let they intersect at M.

By condition,



$PR = PQ$

$$\Rightarrow \left|\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right| = \left|\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right|$$

$$\Rightarrow \boxed{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = \pm\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

Eqn of Angle Bisectors.

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Method to Identify Acute/Obtuse Bisector :

— ~~Consider~~ Calc the angle b/w one of the given lines (L_1 or L_2) & any one of the calcd. angle bisectors (B_1 or B_2).

— Let θ be the angle b/w these 2.

for eg take L_1 & B_1.

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Slope (L_1) = m_1
Slope (B_1) = m_2

if $\tan(\theta) < 1 \Rightarrow$ Acute Bisector
if $\tan(\theta) > 1 \Rightarrow$ Obtuse Bisector

Q) For strt. lines $4x + 3y = 6$ and $5x + 12y + 9 = 0$, find acute & obtuse angle bisectors.

A) B_1 : $\left(\frac{4x + 3y - 6}{5}\right) = \left(\frac{5x + 12y + 9}{13}\right)$

$\Rightarrow 27x - 21y - 123 = 0$

$\Rightarrow \boxed{9x - 7y - 41 = 0}$

B_2 : $$\left(\frac{4x+3y-6}{5}\right) = -\left(\frac{5x+12y+9}{13}\right)$$

$$\Rightarrow \quad 77x + 99y - 33 = 0$$

$$\Rightarrow \quad \boxed{7x + 9y - 3 = 0}$$

Now, $$\tan(\theta) = \left|\frac{(-4/3) - (7/9)}{1 + (-4/3)(7/9)}\right| = \left|\frac{-12-7}{9 - 28/3}\right| = 57$$

$\Rightarrow$ B_1 is Obtuse Bisector

B_2 is Acute Bisector

Q) Find eq^n of st. line thru (4,5) and equally inclined to lines $3x - 4y = 7$ and $5y = 12x + 6$.

A) Angle Bisectors are equally inclined to lines.

$\Rightarrow$ Slope of req. line = Slope of Angle Bisectors.

$$\left(\frac{3x-4y-7}{5}\right) = \left(\frac{12x-5y+6}{13}\right) \Rightarrow m_1 = \left(\frac{-7}{9}\right)$$

$$\Rightarrow m_2 = \left(\frac{9}{7}\right)$$

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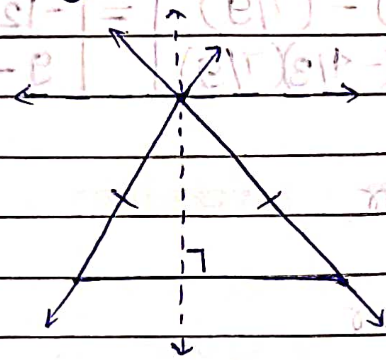
Bisectors

$$\boxed{\begin{aligned} 9y + 7x - 73 &= 0 \\ 7y - 9x + 1 &= 0 \end{aligned}}$$

☆ Angle Bisector Propt in Isoceles △



1 Bisector ∥ to Base

1 Bisector ⊥ to Base

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Position of Pts.

$A(x_1, y_1)$; $B(x_2, y_2)$; $L: ax + by + c = 0$

1) Pts. A & B lie on opp. side of L

$$\boxed{\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right) < 0}$$

$\Rightarrow$ $L(A)$ & $L(B)$ opp. sign.

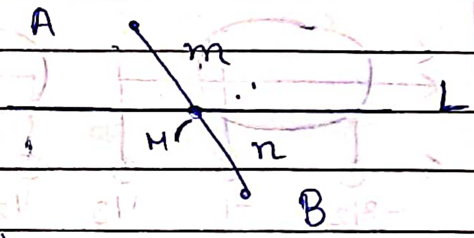
2) Pts. A & B lie on same side of L:

$$\boxed{\left(\frac{ax_1+by_1+c}{ax_2+by_2+c}\right) > 0}$$

$\Rightarrow$ $L(A)$ & $L(B)$ same sign.

Explanation —

$$M \equiv \left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$$



$$\Rightarrow a\left(\frac{nx_1+mx_2}{m+n}\right) + b\left(\frac{ny_1+my_2}{m+n}\right) + c = 0$$

$$\Rightarrow \left(\frac{m}{n}\right) = \frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$$

Internal $(m/n) > 0$

External $(m/n) < 0$

Q) Find exhaustive set of interval of α for which (α, α^2) lies inside Δ having sides $2x+3y=1$, $x+2y-3=0$, $6y=5x-1$.

A) Vertices $\underset{A}{(1/3, 1/9)}$, $\underset{B}{(5/4, 7/8)}$, $\underset{C}{(-7, 5)}$

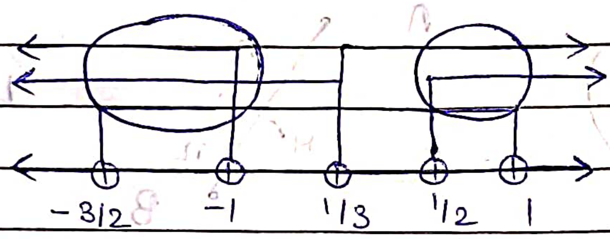
if $BC \equiv x+2y=3$, $AC \equiv 2x+3y=1$, $AB \equiv 6y-5x+$

Since $P(\alpha, \alpha^2)$ inside Δ,

A & P same side of BC $\Rightarrow 2\alpha^2+\alpha-3<0$
$\Rightarrow (2\alpha+3)(\alpha-1)<0$

B & P same side of AC $\Rightarrow 3\alpha^2+2\alpha-1>0$
$\Rightarrow (3\alpha-1)(\alpha+1)>0$

C & P same side of AB $\Rightarrow 6\alpha^2-5\alpha+1>0$
$\Rightarrow (3\alpha-1)(2\alpha-1)>0$



$\boxed{\alpha \in (-3/2, -1) \cup (1/2, 1)}$

Bisector containing a Pt. -

Consider $L_1: a_1x+b_1y+c_1=0$ & $L_2: a_2x+b_2y+c_2=0$
Let $P(\alpha, \beta)$.

- If $(a_1\alpha+b_1\beta+c_1)(a_2\alpha+b_2\beta+c_2)>0$, then bisector containing P,

$$\boxed{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = (+)\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

- If $(a_1\alpha+b_1\beta+c_1)(a_2\alpha+b_2\beta+c_2)<0$, then bisector containing P,

$$\boxed{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = (-)\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

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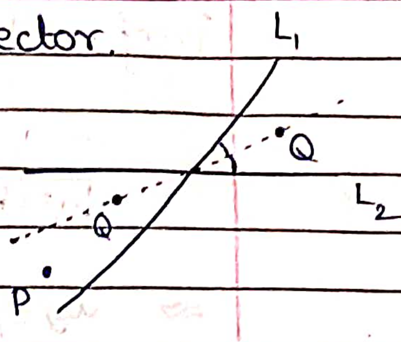
Let WLOG $L_1(P)L_2(P)>0$ pt.

Proof: Let $Q(x,y)$ be locus of req. bisector.



P & Q same side w.r.t. both lines.

OR

P & Q opp. side w.r.t. both lines.

C1: Same side $\Rightarrow$ $L_1(P)L_1(Q)>0$ & $L_2(P)L_2(Q)>0$

for $\angle$ bisector, $\left|\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right| = \left|\dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right|$

$\Rightarrow$ $\left|\left(\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)\right|$ opens with (+ve) if $L_1(P)>0$. Similarly for
" " (−ve) " " <0. other.

C2: Opp. side $\Rightarrow$ $L_1(P)L_1(Q)<0$ & $L_2(P)L_2(Q)<0$

$\Rightarrow$ $|a_1x+b_1y+c_1|$ opens with (+ve) if $L_1(P)<0$. Similarly for
" " (−ve) " " >0. other.

In both cases final eqn with $+$ sign.

Acute / Obtuse Angle Bisector

Let $L_1: a_1x+b_1y+c_1=0$ & $L_2: a_2x+b_2y+c_2=0$
where $c_1, c_2 > 0$ and $a_1b_2 \neq b_1a_2$

Condition	Acute Bisector	Obtuse Bisector
$a_1a_2+b_1b_2>0$	−	+
$a_1a_2+b_1b_2<0$	+	−

Proof: In normal form, lines are,

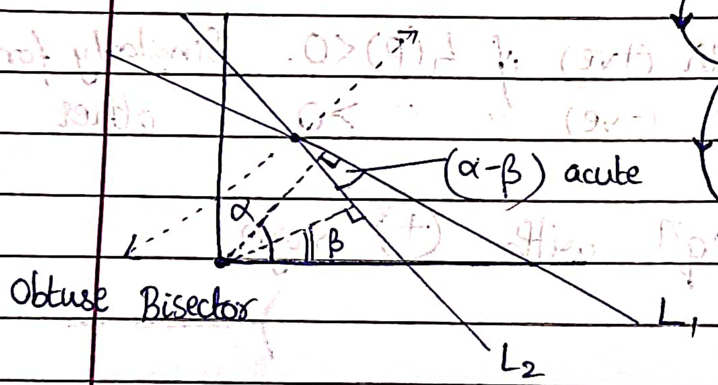
$$\frac{-a_1 x}{\sqrt{a_1^2+b_1^2}} - \frac{b_1 y}{\sqrt{a_1^2+b_1^2}} = \frac{c_1}{\sqrt{a_1^2+b_1^2}} \quad \{c_1 > 0\}$$

$$\Rightarrow c_\alpha = \left(\frac{-a_1}{\sqrt{a_1^2+b_1^2}}\right), \quad s_\alpha = \left(\frac{-b_1}{\sqrt{a_1^2+b_1^2}}\right)$$

Similarly for 2nd Line, $c_\beta = \left(\frac{-a_2}{\sqrt{a_2^2+b_2^2}}\right)$, $s_\beta = \left(\frac{-b_2}{\sqrt{a_2^2+b_2^2}}\right)$.

Let WLOG $\alpha > \beta$. Now, $c_{\alpha-\beta} = \left(\frac{a_1 a_2 + b_1 b_2}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}\right)$

If $(\alpha-\beta)$ acute $\Rightarrow$ $a_1 a_2 + b_1 b_2 > 0$



Obtuse bisector contains origin. Since $c_1, c_2 > 0$

$\Rightarrow$ Obtuse: $\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right) = + \left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)$

Similarly, can be proved for all cases.

Q) Find eq^n of bisector of $\angle$ b/w $x+2y-11=0$ and $3x-6y-5=0$ containing pt. $(1,-3)$.

A) $L_1(1,-3) = 1+2(-3)-11 = (-16) < 0$

$\boxed{L_2(1,-3) = 3(1)-6(-3)-5 = 16 > 0}$

$\Rightarrow$ Bisector with (-ve) sign.

$$B \equiv \left(\frac{x+2y-11}{\sqrt{5}}\right) = -\left(\frac{3x-6y-5}{3\sqrt{5}}\right) \Rightarrow \boxed{x = \left(\frac{19}{3}\right)}$$

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Pair of Strt. Lines

Let $L_1 \equiv a_1x + b_1y + c_1 = 0$ and
$L_2 \equiv a_2x + b_2y + c_2 = 0$.

Joint eq^n of L_1 & L_2 is,

$$(a_1x+b_1y+c_1)(a_2x+b_2y+c_2) = 0 \Rightarrow L_1L_2 = 0$$

This is a Pair of Strt. lines.

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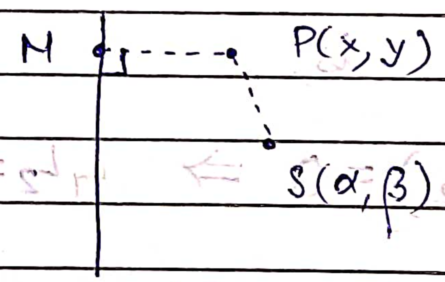
General Eqⁿ of 2nd Degree
(Eqn of General Conic)

General 2nd degree eqⁿ in x, y is

$$\boxed{ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0}$$

where a, h, b, g, f, c are const.

Defⁿ:
- It is locus of a pt. s.t. the ratio of its dist. from a fixed pt. to a fixed line remains const.
- This const. is called eccentricity (e) of conic
- Fixed pt. is known as focus
- Fixed line is known as directrix



$L \equiv a_1x + b_1y + c_1 = 0$

As per defⁿ.

$$\left(\frac{PS}{PM}\right) = \lambda$$

$$\Rightarrow (PS)^2 = \lambda^2 (PM)^2$$

$$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = \lambda^2\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2$$

On simplifying, it will be of form req.

Discriminant :

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\Rightarrow \boxed{\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}}$$

Imp. Results :-

- $\Delta = 0 \Rightarrow$ Eq^n represents Degenerate Conics.

Condition	Nature of Conic
$\Delta=0,\ h^2>ab$	Pair of Real Intersecting Lines
$\Delta=0,\ h^2<ab$	Pair of Imaginary Lines
$\Delta=0,\ h^2=ab$	Coincident Lines

- ~~In terms of eccentricity (e)~~. If $\Delta \neq 0$

Condition	Nature of Conic
$\Delta \neq 0$, $h = 0$, $a = b$	Circle
$\Delta \neq 0$, $h^2 = ab$	Parabola
$\Delta \neq 0$, $h^2 < ab$	Ellipse
$\Delta \neq 0$, $h^2 > ab$	Hyperbola

- In terms of eccentricity (e)

Condition	Nature of Conic
$e = 1$	Parabola
$0 < e < 1$	Ellipse
$e > 1$	Hyperbola
$e = 0$	Circle
$e = \infty$	Pair of St. Lines

Proof: Let us consider it as a quadratic in 'x'.

(1) $ax^2 + x(2hy+2g) + (by^2+2fy+c) = 0$ (D_1)

$$\Rightarrow x = \left(\frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4(by^2+2fy+c)a}}{2a}\right)$$

$\Rightarrow x = Ay + B$ (for this D_1 of orig. quad. should be perfect sq.)

$\Rightarrow (hy+g)^2 = (by^2+2fy+c)a + Q^2$ (D_2)

$\Rightarrow$ (2) $y^2(h^2-ab) + y(2gh-2af) + (g^2-ac) = Q^2$

for this to be perfect sq. $\Rightarrow$ $D_2 = 0$

$\Rightarrow 4(gh-af)^2 = 4(h^2-ab)(g^2-ac)$

$\Rightarrow (gh)^2 + (af)^2 - 2aghf = (hg)^2 - h^2ac - abg^2 + a^2bc$

$\Rightarrow a^2f^2 + h^2ac + abg^2 = 2aghf + a^2bc$

$\Rightarrow abc + 2ghf - h^2c - g^2b - f^2a = 0 \Rightarrow \Delta = 0$

If $h^2 = ab$, (1) has coincident lines as roots (See (2))
If $h^2 < ab$, (2) will be (-ve) of a perfect sq.

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Homogenous Eq^n (in 2 variables)

An eq^n of form

$$a_0 y^n + a_1 y^{n-1} x + a_2 y^{n-2} x^2 + \dots + a_n x^n = 0$$

represents homo. eq^n of x, y of degree 'n'.

∴ wrt general conic eq^n, homo. eq^n in x, y of degree '2' is

$$\boxed{ax^2 + 2hxy + by^2 = 0}$$

Imp Pts :-

1) Homo. eq^n always represents joint eq^n of pair of lines thru origin.

- If $(h^2 > ab)$, the eq^n represents real & distinct lines.
- If $(h^2 = ab)$, the eq^n represents coincident lines
- If $(h^2 < ab)$, the eq^n represents pair of imaginary lines meeting at real pt. $(0, 0)$

Proof: $ax^2 + 2hxy + by^2 = 0$

$\Rightarrow \quad b(y/x)^2 + 2h(y/x) + a = 0$

$\Rightarrow \quad bt^2 + 2ht + a = 0 \qquad \{t = y/x\}$

$\Rightarrow \quad t = \left(\dfrac{y}{x}\right) = \left(\dfrac{-h \pm \sqrt{h^2-ab}}{b}\right)$

If $h^2 - ab > 0 \Rightarrow$ Distinct Lines (inparticular lines thru origin)
If $h^2 - ab = 0 \Rightarrow$ Coincident Lines
If $h^2 - ab < 0 \Rightarrow$ Imaginary Lines

2) Clearly, $ax^2 + 2hxy + by^2 = (y - m_1 x)(y - m_2 x)(K)$

where m_1, m_2 are slopes of lines.

$\Rightarrow \quad \boxed{m_1 + m_2 = \left(\dfrac{-2h}{b}\right)}, \quad \boxed{m_1 m_2 = \left(\dfrac{a}{b}\right)}$

3) Angle b/w Pair of Lines :

$$\boxed{\tan(\theta) = \dfrac{2\sqrt{h^2-ab}}{a+b}}$$

Proof : $|m_1 - m_2| = \sqrt{(m_1+m_2)^2 - 4m_1 m_2}$

$\tan(\theta) = \dfrac{|m_1 - m_2|}{|1 + m_1 m_2|}$

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4) Lines represented by eq^n are $\perp$ if and only if $\boxed{a+b=0}$

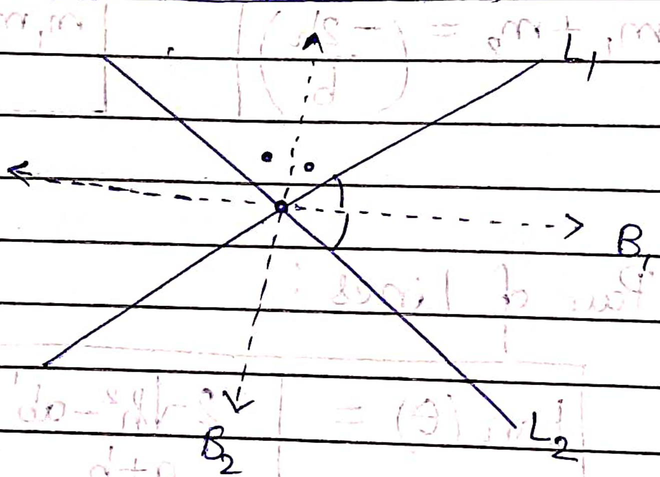
5) Lines represented by eq^n are coincident if and only if $\boxed{h^2=ab}$

6) Pair of lines $\perp$ to lines represented by eq^n is

$$\boxed{bx^2-2hxy+ay^2=0}$$

7) Angle Bisectors :

$$\boxed{\left(\frac{x^2-y^2}{a-b}\right)=\left(\frac{xy}{h}\right)}$$



☆ Angle b/w lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$

is $\boxed{\tan(\theta)=\frac{2\sqrt{h^2-ab}}{a+b}}$

Q) Find eq^n of lines represented by

$$x^2 - 6xy + 8y^2 = 0.$$

Q) If slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is n times the other, then p.t. $4nh^2 = ab(1+n)^2$

Q) P.t. angle b/w lines represented by $(x^2+y^2) s^2_\alpha = (x c_\beta - y s_\beta)^2$ is 2α.

Q) Show that line $y = mx$ bisects angle b/w lines $ax^2 - 2hxy + by^2 = 0$ if $h(1-m^2) + m(a-b) = 0$.

Q) If pair of st. lines $x^2 - 2pxy - y^2 = 0$ & $x^2 - 2qxy - y^2 = 0$ be s.t. each pair bisects the angle b/w other pair, then p.t. $pq = (-1)$.

Q) P.t the eq^n $8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$ represents || lines. Also find ⊥ dist. b/w them.

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A) $8\left(\frac{y}{x}\right)^2 - 6\left(\frac{y}{x}\right) + 1 = 0 \Rightarrow \left(\frac{4y}{x} - 1\right)\left(\frac{2y}{x} - 1\right) = 0$

$\Rightarrow \quad y = x/4 \quad , \quad y = x/2$

A) Let slopes be m & nm.

$m + mn = \left(\frac{-2h}{b}\right) \quad , \quad nm^2 = \left(\frac{a}{b}\right)$

$\Downarrow$

$\Rightarrow m = \frac{-2h}{b(1+n)} \quad \Rightarrow \quad n\left(\frac{-2h}{b(1+n)}\right)^2 = \left(\frac{a}{b}\right)$

$\Rightarrow \quad \boxed{4h^2 n = ab(1+n)^2}$

A) $x^2(s_\alpha^2 - c_\beta^2) + 2 s_\beta c_\beta xy + y^2(s_\alpha^2 - s_\beta^2) = 0$

$\tan(\theta) = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{s_\beta^2 c_\beta^2 - (s_\alpha^2 - c_\beta^2)(s_\alpha^2 - s_\beta^2)}}{(s_\alpha^2 - c_\beta^2) + (s_\alpha^2 - s_\beta^2)}\right|$

$= \left|\frac{2\sqrt{s_\beta^2 c_\beta^2 - s_\alpha^4 + s_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2 - s_\beta^2 c_\beta^2}}{2s_\alpha^2 - 1}\right|$

$= \left|\frac{2 s_\alpha c_\alpha}{2s_\alpha^2 - 1}\right| = \left|t_{2\alpha}\right| \Rightarrow \boxed{\theta = 2\alpha}$

A) Angle Bisectors : $\left(\frac{x^2-y^2}{a-b}\right) = \left(\frac{-xy}{h}\right)$

$\Rightarrow hy^2 - (a-b)xy - hx^2 = 0$

$\Rightarrow ht^2 + (b-a)t - h = 0 \qquad \{t = y/x\}$

has $t = m$ as a sol^n

$\Rightarrow hm^2 + m(b-a) - h = 0 \Rightarrow \boxed{h(1-m^2) + m(a-b) = 0}$

A) Angle Bisectors : $\left(\frac{x^2-y^2}{1-(-1)}\right) = \left(\frac{xy}{-p}\right)$

$\Rightarrow px^2 - py^2 + 2xy = 0 \Rightarrow x^2 + \left(\frac{2}{p}\right)xy - y^2 = 0$

$\equiv x^2 - 2qxy - y^2 = 0$

$\Rightarrow pq = (-1)$

A) $2y^2 + y(8x+13) + (8x^2+26x+15) = 0$

$D = (8x+13)^2 - 4\cdot 2\cdot (8x^2+26x+15)$

$= (64x^2 + 2\cdot 8\cdot 13x + 169) - (64x^2 + 4\cdot 2\cdot 26x + 4\cdot 2\cdot 15)$

$= 49 \Rightarrow \sqrt{D} = 7$

$y = \frac{-(8x+13) \pm 7}{4} \Rightarrow \boxed{2y+4x+3=0}, \boxed{y+2x+5=0}$

$\Rightarrow \boxed{\text{Dist.} = 7/2\sqrt{5}}$

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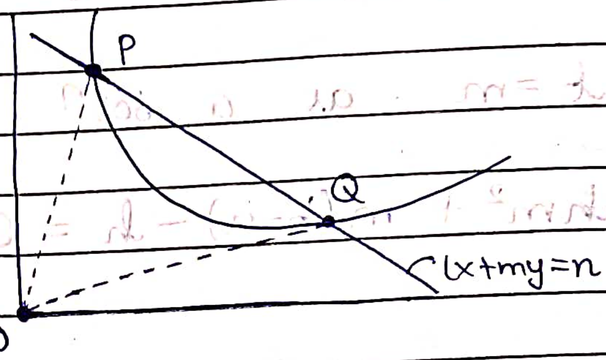
Method of Homogenisation —

Let curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ & line $lx + my - n = 0$ intersect at 2 pts. P & Q.



To find joint eq^n of OP & OQ, we homogenise curve with line.

Process :

— $lx + my = n \Rightarrow \left(\left(\frac{lx+my}{n}\right) = 1\right)$

— $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\Rightarrow ax^2 + 2hxy + by^2 + 2gx(1) + 2fy(1) + c(1^2) = 0$

$\Rightarrow ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{n}\right) + 2fy\left(\frac{lx+my}{n}\right) + c\left(\frac{lx+my}{n}\right)$

On simplifying, $\boxed{Ax^2 + 2Hxy + By^2 = 0}$

This clearly represents joint eq^n of OP & OQ.

Q) P.t. angle b/w lines joining origin to pts. of ∩ of $y = 3x + 2$ with $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

A) $y = 3x + 2 \Rightarrow \boxed{\left(\frac{y-3x}{2}\right) = 1}$

$x^2 + 2xy + 3y^2 + 4x(1) + 8y(1) - 11(1^2) = 0$

$\Rightarrow x^2 + 2xy + 3y^2 + 4x\left(\frac{y-3x}{2}\right) + 8y\left(\frac{y-3x}{2}\right) - 11\left(\frac{y-3x}{2}\right)^2 = 0$

$\Rightarrow 4x^2 + 8xy + 12y^2 + 8x(y-3x) + 16(y-3x)y - 11(y^2 + 9x^2 - 6xy) = 0$

$\Rightarrow x^2(4 - 24 - 99) + y^2(12 + 16 - 11) + xy(8 + 8 - 48 + 66) = 0$

$\Rightarrow 17y^2 + 34xy - 119x^2 = 0$

$\Rightarrow \boxed{y^2 + 2xy - 7x^2 = 0}$

$\Rightarrow \tan(\theta) = \left|\frac{2\sqrt{1^2 - (-7)(1)}}{1 + (-7)}\right| \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$

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Q) Find condition s.t. pair of strt. lines joining origin to $\cap$ of $y = mx + c$ and $x^2 + y^2 = a^2$ may be at right angle

A) $y = mx + c \Rightarrow \boxed{\left(\frac{y - mx}{c}\right) = 1}$

$$x^2 + y^2 = a^2(1)^2 \Rightarrow x^2 + y^2 = a^2\left(\frac{y - mx}{c}\right)^2$$

$$\Rightarrow c^2x^2 + c^2y^2 = a^2y^2 + a^2m^2x^2 - 2a^2mxy$$

$$\Rightarrow x^2(c^2 - a^2m^2) + y^2(c^2 - a^2) + 2a^2mxy = 0$$

for right angle b/w lines,

$$(c^2 - a^2m^2) + (c^2 - a^2) = 0 \Rightarrow \boxed{2c^2 = a^2(1 + m^2)}$$

Q) Show that all chords of $3x^2 - y^2 - 2x + 4y = 0$ which subtends right angle at origin pass thru a fix. pt. Also find that pt.

A) Let req. lines be $y = ax + b$

$$\Rightarrow \left(\frac{y - ax}{b}\right) = 1$$

$$3x^2 - y^2 - 2x + 4y = 0 \Rightarrow 3x^2 - y^2 - 2x\left(\frac{y-ax}{b}\right) + 4y\left(\frac{y-ax}{b}\right) = 0$$

$$\Rightarrow \quad x^2(3b+2a) + y^2(-b+4) + xy(-2-4a) = 0$$

For subtending right angle,

$$(3b+2a) + (4-b) = 0 \Rightarrow \boxed{a+b+2=0}$$

$$\Rightarrow \quad y + (-a)x + (-b) = 0 \longleftrightarrow (-2) + (-a)(1) + (-b)(1) = 0$$

$\Rightarrow$ Passes thru fix pt. $(1, -2)$

Q) Find condition on a & b s.t. portion of $ax + by - 1 = 0$, intercepted b/w $ax + y + 1 = 0$ and $x + by = 0$ subtends right angle at origin.

A) Curve: $(ax + y + 1)(x + by) = 0$

$$\Rightarrow ax^2 + (ab+1)xy + by^2 + x + by = 0$$

Line: $ax + by = 1$

$$\Rightarrow \quad ax^2 + (ab+1)xy + by^2 + x(ax+by) + by(ax+by) = 0$$

$$\Rightarrow \quad x^2(a+a) + xy(ab+1+b+ab) + y^2(b+b^2) = 0$$

$$2ax^2 + (2ab+b+1)xy + b(b+1)y^2 = 0$$

For right angle,

$$\boxed{2a + b + b^2 = 0}$$

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Centre of Curve

Let $f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ be a general conic.

Solve for x, y using following eq^{n}s,

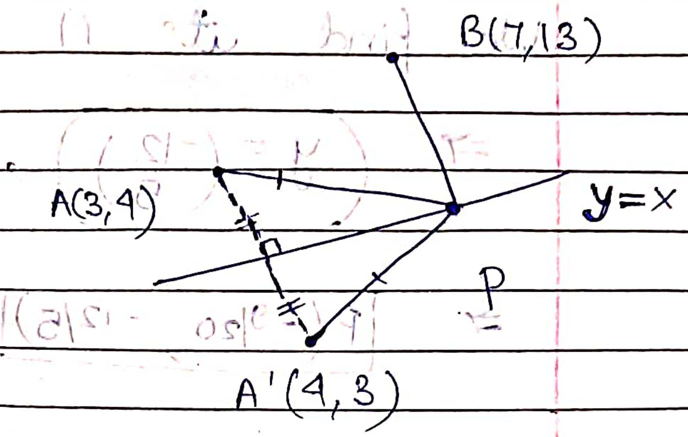
$$\left(\frac{\partial f(x,y)}{\partial y} = 0\right) \quad \& \quad \left(\frac{\partial f(x,y)}{\partial x} = 0\right)$$

(Linear eq^n's in x, y)

The sol^{n}s for x, y give centre of curve

Q) Consider $A(3,4)$ & $B(7,13)$. If P be any pt on $y=x$ s.t. $PA+PB$ is min., then coordinates of P are ?

A) Take image of A in line.



$\Rightarrow$ $A' \equiv (4,3)$

We have, $PA = PA'$

$\Rightarrow PA + PB = PA' + PB \Rightarrow \min(PA+PB) =$ when A', P, B all colliner.

$\Rightarrow P \equiv (y=x) \cap (A'B)$; $A'B \equiv (y-3) = \left(\frac{10}{3}\right)(x-4)$

$\Rightarrow$ $\boxed{P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)}$

Q) Consider $A(0,1)$ & $B(2,0)$. Let P be any pt. on $4x+3y+9=0$. find coordinates of P is $|PA-PB|$ is max. and min.

A) Minimum : If P lies on $\perp$ bisector of AB, then $PA = PB$.

$\perp$ bisector : $y - 2x + \frac{3}{2} = 0$

We find its $\cap$ with $4x + 3y + 9 = 0$

$\Rightarrow \boxed{y = \left(\frac{-12}{5}\right)}, \boxed{x = \left(\frac{-9}{20}\right)}$

$\Rightarrow \boxed{P\left(-9/20, -12/5\right)}$

Maximum : By Δ inequality $|PA - PB| \leq AB$

and equality occurs when P, A, B collinear

AB : $x + 2y - 2 = 0$

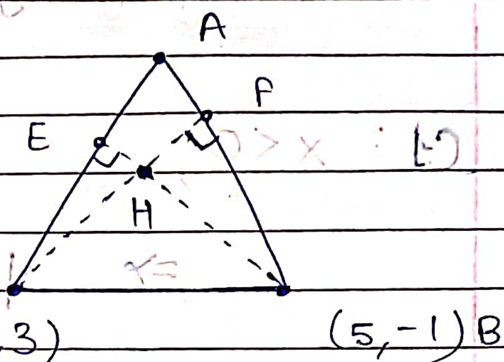
We find its $\cap$ with $4x + 3y + 9 = 0$

$\Rightarrow \boxed{y = \left(\frac{17}{5}\right)}, \boxed{x = \left(\frac{-24}{5}\right)}$

$\Rightarrow \boxed{P\left(-24/5, 17/5\right)}$

Q) 2 vertices of a Δ are $(5,-1)$ and $(-2,3)$. If orthocentre of Δ is origin, then find coordinates of 3rd vertex.

A) $H \equiv (0,0)$



$BH \perp AC$

AC passes thru C

$CF \perp AB$

AB passes thru B.

AC: $y-5x-13=0$ AB: $3y-2x+13=0$

On solving, ~~A(-4,-17)~~ $\boxed{A(-4,-7)}$

Q) For pt. $P(x_1,y_1)$ & $Q(x_2,y_2)$ on coordinate plane, a new dist $d(P,Q)$ is defined by

$$d(P,Q) = |x_1-x_2| + |y_1-y_2|$$

Let $O \equiv (0,0)$ & $A \equiv (3,2)$. P.t. the set of pts. in first quadrant which are equidist. (wrt. new length) from O & A consists of the union of a line segment of finite length and an infinite ray.

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A) Let (x, y) be equidist (wrt new dist) from O & A.

$\Rightarrow \quad |x| + |y| = |x-3| + |y-2|$

C1: $x < 0$, $\quad -x + |y| = 3 - x + |y-2|$

$\Rightarrow \quad |y| = 3 + |y-2|$

Cc1: $y < 0$, $\quad -y = 3 + 2 - y \Rightarrow y \in \phi$

Cc2: $y \in [0, 2)$, $\quad y = 3 + 2 - y \Rightarrow y = 5/2$

Cc3: $y \geq 2$, $\quad y = 3 + y - 2 \Rightarrow y \in \phi$

C2: $x \in [0, 3)$, $\quad x + |y| = (3-x) + |y-2|$

$\Rightarrow \quad |y| - |y-2| = (3-2x)$

Cc1: $y < 0$, $\quad -y - 2 + y = 3 - 2x \Rightarrow x = 5/2$

Cc2: $y \in [0, 2)$, $\quad y - (2-y) = 3 - 2x \Rightarrow 2y + 2x = 5$

Cc3: $y \geq 2$, $\quad y - (y-2) = 3 - 2x \Rightarrow x = 1/2$

C3: $x \geq 3$, $\quad x + |y| = (x-3) + |y-2|$

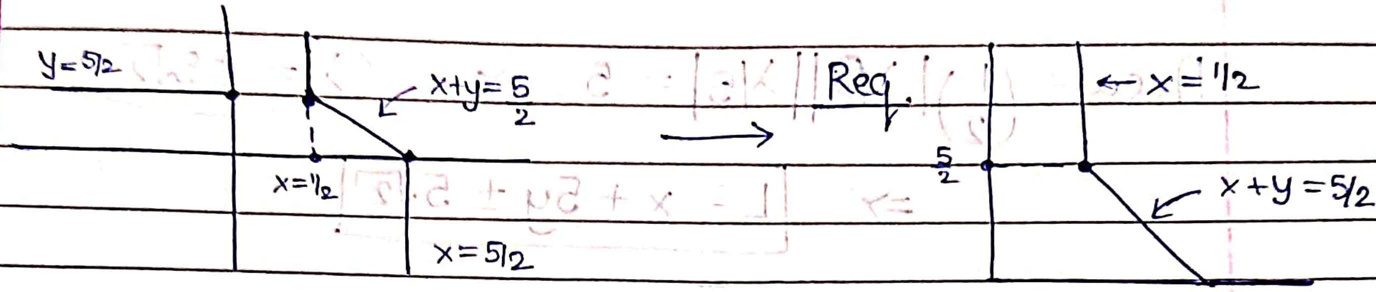
$\Rightarrow \quad |y| = |y-2| - 3$

$\underline{Cc1}$: $y<0$, $\quad -y = 2-y-3 \Rightarrow y \in \emptyset$

$\underline{Cc2}$: $y \in [0,2)$, $\quad y = (2-y)-3 \Rightarrow y = -1/2$

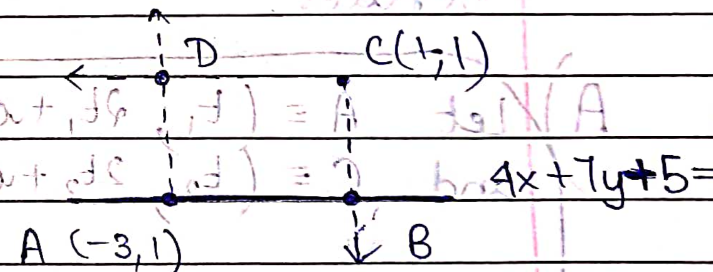
$\Rightarrow y \in \emptyset$

$\underline{Cc3}$: $y \geq 2$, $\quad y = (y-2)-3 \Rightarrow y \in \emptyset$



Q) One side of a rect. lies along $4x+7y+5=0$. 2 of its vertices are $(-3,1)$ & $(1,1)$. Find other 3 sides.

A) On plotting,



AD ⊥ Line,

$\Rightarrow \boxed{7x-4y+25=0}$

CB ∥ AD, $\Rightarrow \boxed{7x-4y-3=0}$

CD ∥ Line, $\Rightarrow \boxed{4x+7y-11=0}$

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Q) A strt. line L is $\perp$ to $5x - y = 1$. The area of Δ formed by L and coordinate axes is 5. Find eqn of L.

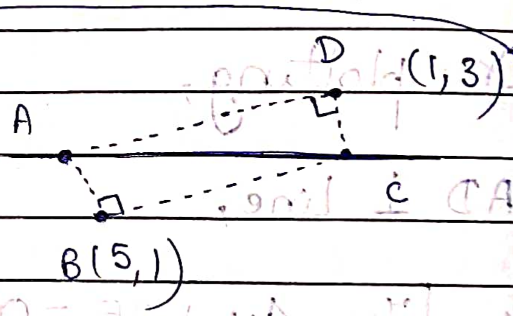
A) Let $L \equiv x + 5y - \lambda = 0 \Rightarrow \dfrac{x}{(\lambda)} + \dfrac{y}{(\lambda/5)} = 1$

$$\text{Area} = \left(\frac{1}{2}\right)|\lambda||\lambda/5| = 5 \Rightarrow \lambda = \pm 5\sqrt{2}$$

$$\Rightarrow \boxed{L \equiv x + 5y \pm 5\sqrt{2}}$$

Q) The pt. (1, 3) and (5, 1) are 2 opp. vertices of a rect. The other 2 vertices lie on $y = 2x + c$. Find c and remaining vertices.

A) Let $A \equiv (t_1, 2t_1 + c)$
and $C \equiv (t_2, 2t_2 + c)$



We have,

AB $\perp$ BC and AD $\perp$ DC and AD $\parallel$ BC

$$\Rightarrow \left(\frac{2t_1 + c - 1}{t_1 - 5}\right) = \cancel{\frac{-1}{2}} \left(\frac{5 - t_2}{2t_2 + c - 1}\right), \quad \left(\frac{2t_2 + c - 3}{t_2 - 1}\right) = \left(\frac{1 - t_1}{2t_1 + c - 3}\right),$$

$$\left(\frac{2t_1 + c - 3}{t_1 - 1}\right) = \left(\frac{2t_2 + c - 1}{t_2 - 5}\right)$$

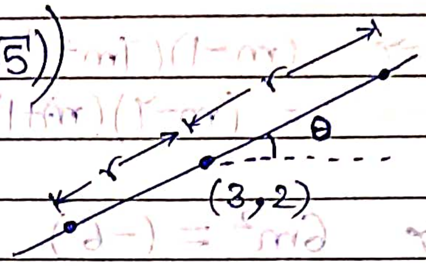
Midpt. of (1,3) & (5,1) lies on $y = 2x + c$.

$\Rightarrow \left(\frac{1+3}{2}\right) = 2\left(\frac{5+1}{2}\right) + c \Rightarrow \boxed{c = (-4)}$

Length of diag. same $\Rightarrow$ $r = \sqrt{5}$

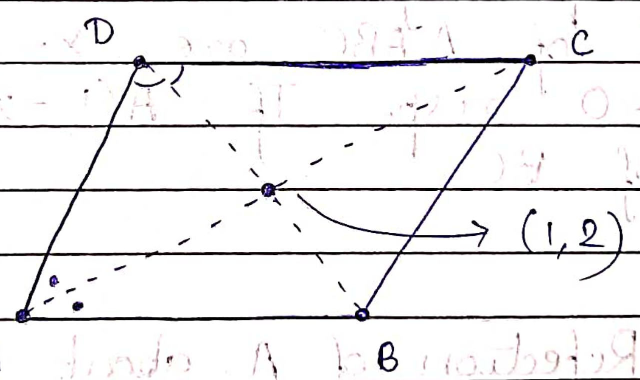
$\text{Vertex}_1 = \left(3 \pm \sqrt{5}\left(1/\sqrt{5}\right),\ 2 \pm \sqrt{5}\left(2/\sqrt{5}\right)\right)$

$\Rightarrow \boxed{(4,4)}\ ;\ \boxed{(2,0)}$



Q) ☆ 2 sides of a rhombus ABCD are $\parallel$ to lines $y = x+2$ & $y = 7x+3$. If diags. $\cap$ at (1, 2) and vertex A is on Y axis, then find possible coordinates of A.

A) ☆ (Diags. are $\angle$ bisectors of sides)



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Let 'm' be slope of diags.

Since sides $\parallel$ to $y=x+2$ & $y=7x+3$

$\Rightarrow \left|\frac{m-1}{1+m}\right| = \left|\frac{m-7}{1+7m}\right| \Rightarrow (m-1)^2(7m+1)^2 = (m-7)^2(m+1)^2$

$\Rightarrow (m-1)(7m+1) = (m-7)(m+1)$ OR $(m-1)(7m+1)+(m-7)(m+1) = 0$

$\Rightarrow 6m^2 = (-6)$ OR $8m^2-12m-8=0$

$\Rightarrow$ No sol^n $\Rightarrow m = 2, -1/2$

$\Rightarrow D_1 : y-2x=0$, $D_2 : 2y+x-5=0$

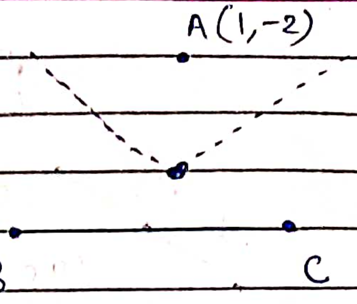
Since A lies on diag.

$\Rightarrow \boxed{A \equiv (0,0) ; (0, 5/2)}$

Q) The eq^{n}s of $\perp$ bisectors of sides AB and AC of ΔABC are $x-y+5=0$ and $x+2y=0$ resp. If $A(1,-2)$, then find eq^n of BC.

A) ☆ Refection of A about AB & AC's $\perp$ bisector gives B & C

$$\left(\frac{x_B-1}{1}\right)=\left(\frac{y_B+2}{-1}\right)=(+2)\left(\frac{1+2+5}{2}\right)$$



$\Rightarrow$ $\cancel{B(9)}$ $B(-7,6)$

$$\left(\frac{x_C-1}{1}\right)=\left(\frac{y_C+2}{2}\right)=(-2)\left(\frac{1-4}{5}\right) \Rightarrow C(11/5,\ 2/5)$$

$\Rightarrow$ $\boxed{BC:\ 23y+14x\ \underset{40}{\cancel{\oplus\,32}}=0}$

Q) Lines $L_1: ax+by+c=0$, $L_2: lx+my+n=0$ intersect at pt. P and make angle θ with each other. Find eq^n of line, diff. from L_2, which passes thru P and makes angle θ with L_1.

A) Let $L: (a+\lambda l)x+(b+\lambda m)y+(c+\lambda n)=0$.

We have, $\tan(\theta)=\left|\dfrac{-\left(\frac{a+\lambda l}{b+\lambda m}\right)+\left(\frac{a}{b}\right)}{1-\left(\frac{a}{b}\right)\left(\frac{a+\lambda l}{b+\lambda m}\right)}\right|=\left|\dfrac{-\frac{l}{m}+a/b}{1-al/bm}\right|$

$$\Rightarrow \left|\frac{a(b+\lambda m)-b(a+\lambda l)}{b(b+\lambda m)-a(a+\lambda l)}\right|=\left|\frac{am-bl}{bm-al}\right|$$

$$\Rightarrow \left|\frac{\lambda(am-bl)}{(b^2-a^2)+\lambda(bm-al)}\right|=\left|\frac{am-bl}{bm-al}\right| \Rightarrow |\lambda(bm-al)| = |(b^2-a^2)+\lambda(bm-al)|$$

$$\Rightarrow \boxed{\lambda=\left(\frac{a^2-b^2}{2(bm-al)}\right)}$$

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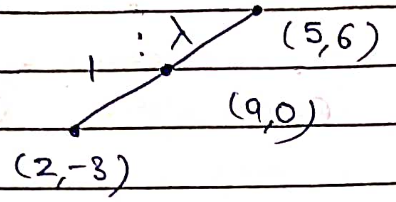
Exercise 2 (Part 1)

Q1)



$$\left(\frac{6-3\lambda}{\lambda+1}\right) = 0 \Rightarrow \lambda = 2$$

$\Rightarrow$ 2:1 internally

Q2)

$$13x + 11(mx-1) = 700 \Rightarrow x = \left(\frac{711}{11m+13}\right) \in \mathbb{Z}$$

$$\Rightarrow (11m+13) \mid 711$$

$$\Rightarrow 11m+13 \in \{-711, -79, -9, -1, 1, 9, 79, 711\}$$

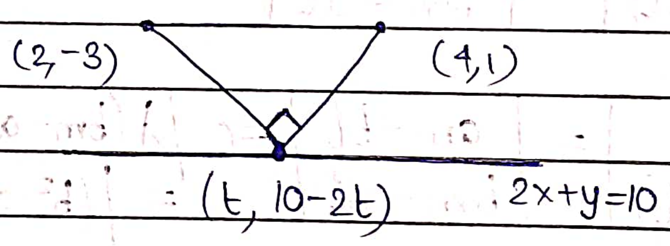
$$\Rightarrow m \in \left\{\frac{-724}{11}, \frac{-92}{11}, -2, \frac{-14}{11}, \frac{-12}{11}, \frac{-4}{11}, 6, \frac{698}{11}\right\}$$

$$m \in \mathbb{Z}^+ \Rightarrow m = 6$$

Q3)

$$m_1 + m_2 = 4m_1m_2 \Rightarrow \left(\frac{2c}{-7}\right) = 4\left(\frac{1}{-7}\right) \Rightarrow c = 2$$

Q4)



$$\left(\frac{9-2t}{t-4}\right)\left(\frac{13-2t}{t-2}\right) = (-$$

$$\Rightarrow (2t-9)(2t-13) + (t-2)(t-4)$$

$$\Rightarrow 5t^2 - 50t + 125 = 0$$

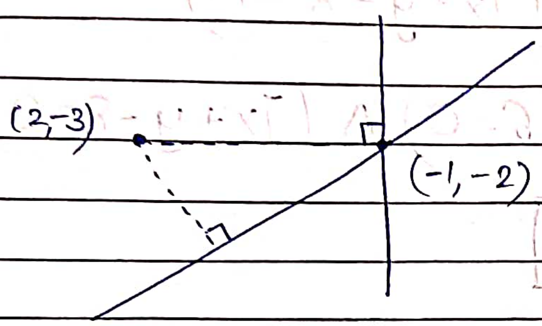
$$\Rightarrow (t-5)^2 = 0 \Rightarrow t = 5 \Rightarrow \text{Pt. } (5,0)$$

$$\text{Area of } \Delta = \left(\frac{1}{2}\right)\left\|\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ -3 & 1 & 0 \end{vmatrix}\right\| = \left(\frac{1}{2}\right)\left\|\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 3 \\ -3 & 4 & 3 \end{vmatrix}\right\|$$

$\Rightarrow$ $\boxed{\text{Area} = 3}$

Q5) $2x + y + 4 = 0$ $\Rightarrow$ Pt. $(-1, -2)$

$x - 2y - 3 = 0$



Max. dist. of any line passing thru $(-1,-2)$ from $(+2,-3)$

$= \sqrt{10}$

$\Rightarrow$ 1 such line exists.

Q6) AB: $y = 0$; BC: $x + y = 8$; CA: $3x - 4y + 12 = 0$

$\Rightarrow$ C $(20/7, 36/7)$; $A(-4, 0)$; $B(8, 0)$

A & $(0, \beta)$ same side of BC, $(\beta - 8)(-4 - 8) > 0$

$\Rightarrow \beta < 8$

B " " " " " AC,

$(12 - 4\beta)(24 + 12) > 0 \Rightarrow \beta < 3$

C " " " " " AB, $\beta > 0$

$\Rightarrow$ $\boxed{\beta \in (0, 3)}$

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Q7) $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow \quad \beta - 1 = 0 \quad \Rightarrow \quad \beta = 1, \ \alpha = \pm 1$

Q8) $(x - 7y + 6 = 0) \perp (7x + y - 8 = 0)$

$\Rightarrow$ H is $(x - 7y + 6 = 0) \wedge (7x + y - 8 = 0)$

$\Rightarrow \quad \boxed{H \equiv (1, 1)}$

Q9) Let chord be $lx + my = 1$.

By Homogenization, $2x^2 + 3y^2 - 5x(lx + my) = 0$

$\Rightarrow \quad (2 - 5l)x^2 - 5mxy + 3y^2 = 0$

for right angle at origin,

$(2 - 5l) + 3 = 0 \Rightarrow l = 1 \Rightarrow$ Chord: $x + my = 1$

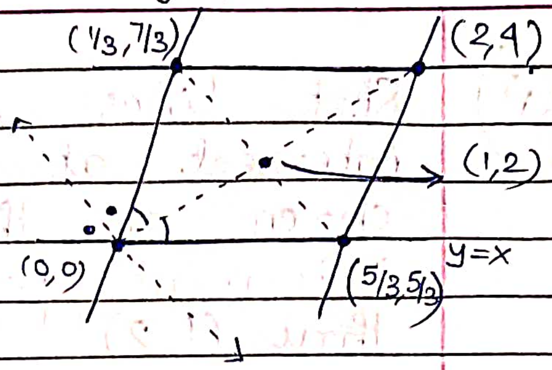
$\Rightarrow$ $\boxed{\text{Fix. Pt. } (-1, 0)}$

Q10)

☆ $\boxed{\text{Diags. } \parallel \text{ to } \angle \text{ bisectors of Sides in a rhombus.}}$

$$\left(\frac{y-x}{\sqrt{2}}\right) = \pm\left(\frac{y-7x}{5\sqrt{2}}\right)$$

$\downarrow$ $\boxed{m = 2}$ $\qquad$ $\downarrow$ $\boxed{m = \left(-\tfrac{1}{2}\right)}$

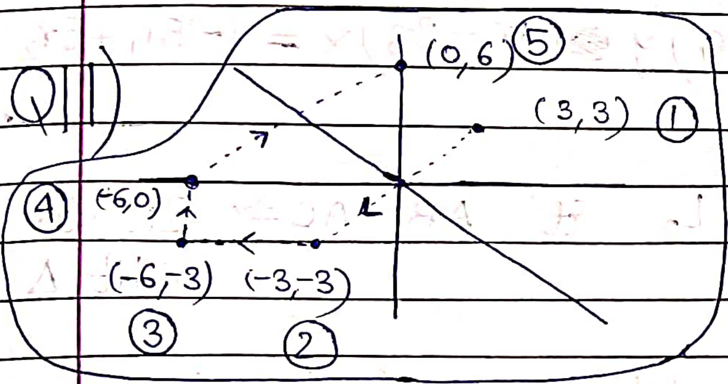


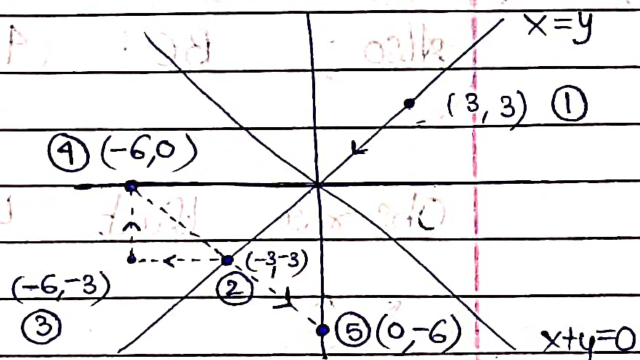
$\Rightarrow$ $D_1 : \; y = 2x \qquad D_2 : \; 2y + x = 5$

By $\cap$ of lines & Midpt. Theorem, find vertices.

$$\text{Area} = \left(\frac{1}{2}\right) d_1 d_2 = \left(\frac{1}{2}\right)\left(2\sqrt{5}\right)\left(\frac{2\sqrt{5}}{3}\right) \Rightarrow \boxed{\text{Area} = 10/3}$$

Q11)





$(3,3) \xrightarrow[x=(-y)]{} (-3,-3) \xrightarrow[x \to x-3]{} (-6,-3)$

$\downarrow y \to y+3$

Final Pt. $(0,-6)$ $\qquad$ $(0,-6) \xleftarrow[x=y]{} (-6,0)$

Q) ☆ Strt. lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at A. B & C pts. are chosen on these 2 lines s.t. $AB = AC$. Determine possible eq^{n}s of BC passing thru $(1, 2)$

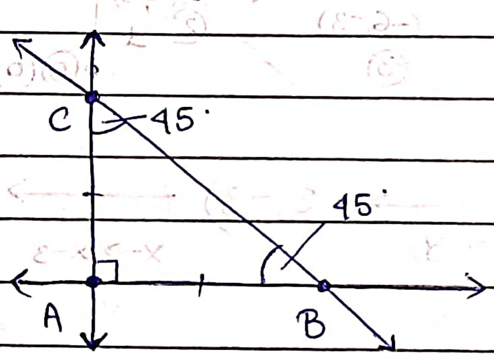
A) We have A $(3, -1)$.

Let $B\left(3 + r_1(3/5),\ -1 + r_1(4/5)\right)$ and $C\left(3 + r_2(-4/5),\ -1 + r_2(3/5)\right)$

We have, $|r_1| = |r_2|$

Also, BC: $(4r_2 + 3r_1)y \oplus (4r_1 - 3r_2)x = (-15r_1 + 5r_2)$

Observe that $L_1 \perp L_2$ & $AB = AC \Rightarrow$ Isosceles right Δ



Let slope of BC 'm'.

$$\left|\frac{m - (-3/4)}{1 - 3m/4}\right| = 1 \Rightarrow m = (-7), \frac{1}{7}$$

$$\left|\frac{m - 4/3}{1 + 4m/3}\right| = 1 \Rightarrow m = (-7), \frac{1}{7}$$

Lines

$y + 7x - 9 = 0$

$7y - x - 13 = 0$

Q) A line L with (-ve) slope passes thru (8,2) and cuts (+ve) coordinate axes at P & Q. Find min. value of $|OP + OQ|$ as L varies, where O is origin.

A) $L: \quad y + mx = (8m+2) \qquad \{m > 0\}$

$$\Rightarrow \frac{y}{(8m+2)} + \frac{x}{(8+2/m)} = 1$$

Let $P(8+2/m, 0)$ and $Q(0, 8m+2)$.

$$|OP + OQ| = \left|8 + \frac{2}{m} + 8m + 2\right| = \left|18 + 2\left(2\sqrt{m} + \frac{1}{\sqrt{m}}\right)^2\right|$$

$$\Rightarrow \boxed{|OP + OQ| \geq 18}$$

☆ Q) The eqns of sides AB, BC, AC of a Δ are

$3x + 4y = 6$, $12x - 5y = 3$, $4x - 3y + 12 = 0$.

Find eqn of internal $\angle$ bisector of A.

☆ Q) In ΔABC, $A(4,-1)$ and eqn of $\angle$ bisectors thru B & C are $x + y = 1$ and $2x - y + 2 = 0$. (not necessarily both internal). Find eqn of line BC.

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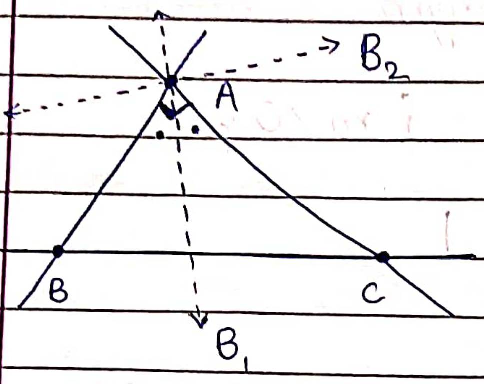
A) AB: $3x+4y=6$, BC: $12x-5y=3$, AC: $4x-3y+12=0$

~~A(39/4,~~ $C\left(\frac{69}{16}, \frac{39}{4}\right)$; $B\left(\frac{2}{3}, 1\right)$



☆ B_1 internal bisector of A

⇒ B & C ~~same~~ opp. side of B_1

$$\left(\frac{3x+4y-6}{5}\right) = \pm\left(\frac{4x-3y+12}{5}\right)$$

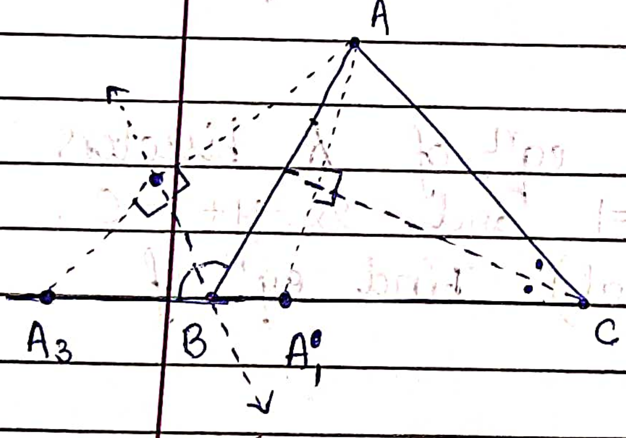
B_1: $x-7y+18=0$ B_2: $7x+y+6=0$

↓ ↓

Opp. side Same side

⇒ $\boxed{\text{Internal Bisector: } x-7y+18=0}$

A) ☆ Reflection of A about internal angle bisectors lies on BC.



$$\left(\frac{x_{A_1}-4}{1}\right) = \left(\frac{y_{A_1}+1}{1}\right) = (-2)\left(\frac{4-1-1}{2}\right)$$

⇒ ~~$A_1(2,-1)$~~ $A_1(2,-3)$

$$\left(\frac{x_{A_2}-4}{2}\right) = \left(\frac{y_{A_2}+1}{-1}\right) = (-2)\left(\frac{8+1+2}{5}\right)$$

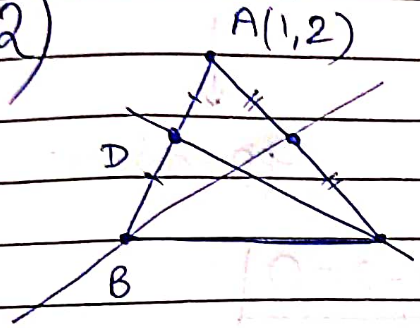
⇒ $A_2\left(-\frac{24}{5}, \frac{17}{5}\right)$

Since A_1, A_2 lie on $BC \Rightarrow Eq^n(BC) = Eq^n(A_1A_2)$

$\Rightarrow \boxed{BC: 16x+17y+19=0}$

Exercise 2 (Part 2)

Q12)

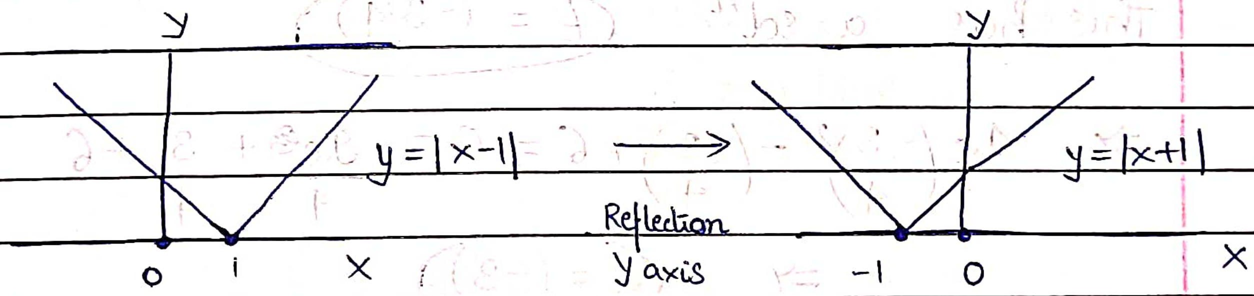


Midpt of AB lies on Median thru C.

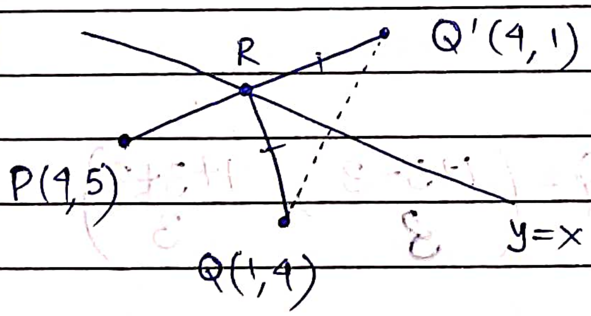
$B(t, 5-t) \Rightarrow D \equiv \left(\frac{t+1}{2}, \frac{7-t}{2}\right)$

$\Rightarrow \left(\frac{t+1}{2}\right) = 4 \Rightarrow \boxed{B(7,-2)}$

Q13)



Q14)

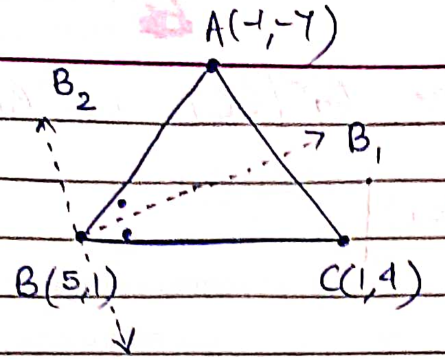


PR + RQ min. $\Rightarrow$ P, R, Q' collinear.

$\Rightarrow R \equiv (y=x) \cap (x=4)$

$\Rightarrow \boxed{R(4,4)}$

Q15)



A & C on opp. sides of internal bisector of $\angle ABC$

$$\left(\frac{4y+3x-19}{5}\right) = \pm\left(\frac{3y-4x+17}{5}\right)$$

$\Rightarrow B_2:\ y+7x-36=0$ $\downarrow$ Same side

$B_1:\ 7y-x-2=0$ $\downarrow$ Opp. side

$\Rightarrow$ $\boxed{\text{Internal Bisector: } 7y-x-2=0}$

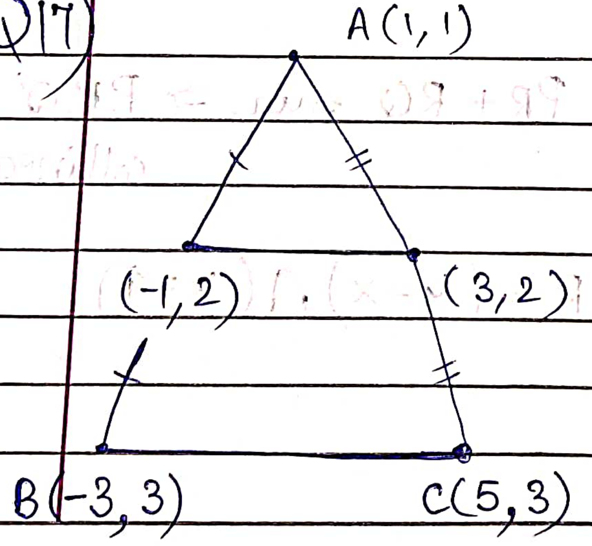
Q16) Let $t = y/x \Rightarrow$ Eqn: $ct^2 - t + 6 = 0$

This has a solⁿ, $t = (-3/4)$

$$\Rightarrow c\left(\frac{-3}{4}\right)^2 - \left(\frac{-3}{4}\right) + 6 = 0 = \frac{9c}{4} + \frac{3}{4} + 6$$

$\Rightarrow c = (-3)$

Q17)



$$G \equiv \left(\frac{1+5-3}{3}, \frac{1+3+3}{3}\right)$$

$\Rightarrow \boxed{G \equiv (1, 7/3)}$

Q18) Let $P(2+t,\ 2-t) \Rightarrow \left(\begin{array}{c}\perp \text{ dist of} \\ P \text{ from } 4x+3y=10\end{array}\right) = 1$

$\Rightarrow \left|\dfrac{4(2+t)+3(2-t)-10}{5}\right| = 1 \Rightarrow |t+4| = 5 \Rightarrow t = 1, (-9)$

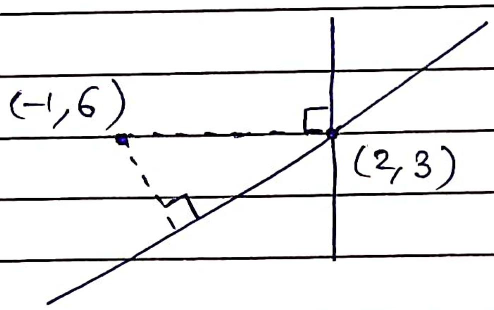
$\Rightarrow \boxed{P_1(3,1)}$ and $\boxed{P_2(-7,11)}$

Q19)

$$\text{Area} = \left(\frac{1}{2}\right)\left\|\begin{array}{cc} 1 & 1 \\ 7 & -3 \\ 12 & 2 \\ 7 & 21 \\ 1 & 1 \end{array}\right\| = \left(\frac{1}{2}\right)\left|\begin{array}{c} -10+14+36+252 \\ -14+7-21 \end{array}\right|$$

$\Rightarrow \boxed{\text{Area} = 132}$

Q20)



$\left(\begin{array}{l}\text{Max. dist.} \\ \text{of any line} \\ \text{thru } (2,3) \text{ from } (-1,6)\end{array}\right) = 3\sqrt{2} < 6$

$\Rightarrow$ 0 lines exist

Q21) AB : $y = x$; BC : $x+y=0$; AC : $2x+3y=6$

$\Rightarrow$ C : $(-6,6)$; A : $(6/5,\ 6/5)$; B : $(0,0)$

C & $(-2,a)$ same side of AB $\Rightarrow a-(-2) > 0 \Rightarrow a > (-2)$
B " " " " " AC $\Rightarrow (-10)+3a < 0 \Rightarrow a < 10/3$
A " " " " " BC $\Rightarrow a+(-2) > 0 \Rightarrow a > 2$

$\Rightarrow \boxed{a \in (2,\ 10/3)}$